Courses of Studies:
UNIT - I

## Statistical Data and Descriptive Statistics (With the use of Excel and Other Statistical Software)

Nature and Classification of data: Univariate, Bivariate and Multivariate data; time-series and cross-sectional data.

## Measure of Central Tendency:

1. Mathematical averages including Arithmetic mean, Geometric mean and Harmonic mean, Properties and applications.
2. Positional Averages

More and Median and other partition values including Quartiles, Deciles and Percentiles.

## UNIT - II

## Measures of Variation (With the use of Excel and Other Statistical Software)

Absolute and relative, Range, Quartile Deviation, Mean Deviation, Standard Deviation, and their coefficients, Properties of Standard Deviation/Variance

Skewness: Meaning, Measurement using Karl Pearson and Bowley's measures; Concept of Kurtosis.

## UNIT - III

## Simple Correlation and Regression Analysis (With the use of Excel and Other Latest Software)

Correlation Analysis: Meaning of Correlation: Simple, multiple and partial; linear and nonlinear correlation and Causation; Scatter diagram, Peasons's coefficient of Correlation, calculation and properties (proofs not required) Correlation and Probable error, Rank Correlation.

Regression Analysis: Principle of least square and Regression lines, Regression equations and estimation, Properties of regression coefficients, Relationship between Correlation and Regression coefficient, Standard error of estimate.

UNIT - IV

## Index Numbers (With use of Excel and Other Latest Software)

Meaning and uses of index number. Construction of index numbers: fixed and chain base; univariate and composite, Aggregative and average of relatives - simple and weighted.

Test of adequacy of index numbers. Base shifting, splicing and deflating, Problems in the construction of index numbers.

Construction of consumer price indices, important share price indices.

## Time Series Analysis (With use of Excel and Other Latest Software)

Components of time series, Additive and multiplicative models, Trend analysis. Fitting of trend line using the principle of least squares - linear, second degree parabola and exponential. Conversion of annual linear trend equation to quarterly/monthly basis and vice-versa. Moving averages, Seasonal variation - Calculation of Seasonal Indices using Simple averages, Ratio-totrend, and Ratio-to-moving average methods. Uses of seasonal indices.

4. The mean of the following frequency distribution is 50 . But the frequencies $f_{1}$ and $f_{2}$ in classes $20-40$ and $60-80$ are missing. Find the missing frequencies:

| Class : | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency: | 17 | $f_{1}$ | 32 | $f_{2}$ | 19 | Total 120 |

Ans: $f_{1}=28, f_{2}=24$
5. 100 students appeared for an examination. The result of those who failed are given below:

| Marks | 5 | 10 | 15 | 20 | 25 | 30 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 4 | 6 | 8 | 7 | 3 | 2 | 30 |

If the average marks of all students were 68.6, find out average marks of those who passed.
Ans: 91.21
6. Out of 50 examinees, those passing the examination are shown below. If the average mark of all the examinees is 5.16 , what would be the average marks of examinees having failed in it?

| Marks obtained | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students passing the Exam. | 8 | 10 | 9 | 6 | 4 | 3 |

Ans: 2.1
7. Define a 'weighted mean'. Under what circumstances would you prefer it to an unweighted mean?
Calculate the weighted mean price of a table from the following data, assuming that weights are proportional to the number of tables sold:

| Price per table (Rs) | 3,600 | 4,000 | 4,400 | 4,800 |
| :--- | :---: | :---: | :---: | :---: |
| No. of tables sold | 14 | 11 | 9 | 6 |

Ans: Rs 4,070
8. 26. Compute the weighted arithmetic mean of the index number from the data below:

|  | Group |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Index No. | Food | Clothing | Fuel and Light | House Rent | Miscellaneous |
| Weights | 125 | 133 | 141 | 173 | 182 |

Ans: 141.15
9. The following table gives the distribution of 100 accidents during seven days of the week of a given month. During the particular month there are 5 Mondays, Tuesday s and Wednesday s and only four each of the other days. Calculate the average number of accidents per day:

| Days | No. of Accidents | Days | No. of Accidents |
| :--- | :---: | :--- | :---: |
| Sunday | 26 | Thursday | 8 |
| Monday | 16 | Friday | 10 |
| Tuesday | 12 | Saturday | 18 |
| Wednesday | 10 |  |  |

Ans: $14.13=14$
10. To produce a scooter of a certain make, labour of different kinds is required in quantities as follows:

| Skilled labour | $:$ | 50 hours |
| :--- | :--- | ---: |
| Semi-skilled labour | $:$ | 100 hours |
| Unskilled labour | $:$ | 300 hours |

If hourly wage rates for these three kinds of labour are Rs 100, Rs 70 and Rs 20 respectively, what is the average labour cost per hour in producing the scooter?
Hints: Use weighted arithmetic mean
Ans: Rs $\mathbf{4 0}$ per hour
11. A candidate obtained the following percentage of marks in different subjects in the HalfYearly Examination:

| English | Statistics | Cost Accountancy | Economics | Income Tax |
| :---: | :---: | :---: | :---: | :---: |
| $46 \%$ | $67 \%$ | $72 \%$ | $58 \%$ | $53 \%$ |

It is agreed to give double weights to marks in English and Statistics as compared to other subjects. What is the simple and weighted arithmetic mean?
Ans: Simple A.M. = 59.2\%; Weighted A.M. = 58.43\%
12. Calculate simple and weighted arithmetic averages from the following data and comment on them:

| Designation | Daily salary (in Rs) | Strength of the cadre |
| :--- | :---: | :---: |
| Class I Officer | 1,500 | 10 |
| Class II Officer | 800 | 20 |
| Subordinate staff | 500 | 70 |
| Clerical staff | 250 | 100 |
| Lower staff | 100 | 150 |

Ans: Simple A.M. = Rs 630; Weighted A.M. = Rs 302.86
13. Comment on the performance of the students of three Universities given blow using an appropriate average:

| University | A |  | B |  | C |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Courses of Study | \% of Pass | No. of <br> students in <br> hundred   | $\begin{array}{ll} \hline \% & \text { of } \\ \text { Pass } \end{array}$ | No. of students in hundred | \% of Pass | No. of studen in hundred |
| M.A. | 81 | 2 | 82 | 2 | 71 | 3 |
| M. Com. | 76 | 3.5 | 76 | 3 | 83 | 4 |
| M.Sc. | 73 | 2 | 60 | 7 | 66 | 3 |
| B. Com. | 58 | 2 | 76 | 7 | 74 | 2 |
| B. Sc. | 70 | 7 | 65 | 3 | 65 | 3 |
| B.A. | 74 | 4.5 | 73 | 6 | 73 | 5 |

Ans: Simple average (A.M.) of pass percentage is $\mathbf{7 2 \%}$ in each case, we are unable to distinguish between performance of students in the three universities.
However, on the basis of weighted average of pass percentage, University $\mathbf{C}$ (72.55\%) is the best followed by University A (72.05\%) and University B (70.61\%)
14. From the results of two colleges A and B given below, state which of them is better and why?

| Name of Examination | College A |  | College B |  |
| :--- | :---: | :---: | :---: | :---: |
| M.A | 60 | 50 | 200 | 160 |
| M. Com. | 100 | 90 | 240 | 190 |
| B.A. | 400 | 300 | 200 | 140 |
| B. Com. | 240 | 150 | 160 | 100 |
| Total | 800 | 590 | 800 | 590 |

(4)

Hints and Ans: Find the weighted average of percentage of passed students (X), the corresponding weights $(W)$ being the number of students appeared. Both the colleges have the same average i.e. $\mathbf{7 3 . 7 5 \%}$.
15. A travelling salesman made five trips in two months. The record of sales is given below: * The sales manager criticized the salesman's * performance as not very good since his mean 4 daily sales were only Rs $54,000(2,70,000 / 5)$. The salesman called this an unfair statement for his daily mean sales were as high as Rs 55,200 (13,80,000/25). What does each average mean here? Which average seems to be more appropriate?

| Trip | No. of <br> days | Value of <br> sales(in'00 Rs) | Sales per day <br> (in'00 Rs) |
| :---: | :---: | :---: | :---: |
| 1 | 5 | 3,000 | 600 |
| 2 | 4 | 1,600 | 400 |
| 3 | 3 | 1,500 | 500 |
| 4 | 7 | 3,500 | 500 |
| 5 | 6 | 4,200 | 700 |
|  | 25 | 13,800 | 2,700 |

Ans: The Manager obtained the simple arithmetic mean of the sales per day, while the salesman obtained the weighted arithmetic mean. The latter (weighted average) seems to be more appropriate.

## Exercise B (Partition Values)

1. Find the median of the following two series:

| (i) | 38 | 34 | 39 | 35 | 32 | 31 | 37 | 30 | 41 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| (ii) | 30 | 31 | 36 | 33 | 29 | 28 | 35 | 36 |  |

Ans: (i) 35 (ii) 32
2. Following are the marks obtained by a batch of 10 students in a certain class test in Statistics (X) and Accountancy (Y).

| Roll No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X$ | 63 | 64 | 62 | 32 | 30 | 60 | 47 | 46 | 35 | 28 |
| $Y$ | 68 | 66 | 35 | 42 | 26 | 85 | 44 | 80 | 33 | 72 |

In which subject is the level of knowledge of the students is higher?
Ans: Med $(X)=46.5$; Med $(Y)=55$. Level of knowledge of students is higher in Accountancy.
3. Find mean and median from the data given below:

| Marks obtained | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 12 | 18 | 27 | 20 | 17 | 6 |

Ans: Mean = 28; Median = 27.41
4. Calculate arithmetic mean and median from the following series:

| Income (Rs) | $0-5$ | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 7 | 10 | 8 | 6 | 4 |

[C.S. (Foundation) Dec. 2000]
Ans: Arithmetic mean = 14.375; Median = 14
5. From the data given below, find the missing frequency, if the Arithmetic Mean is Rs 33. Also find the median of the series:

| Loss per shop (Rs) | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of shops | 10 | 15 | 30 | ----- | 25 | 20 |

[C.A. (Foundation) Nov. 2000]
Ans: Missing frequency = 25; Median = 33
6. Given below is the distribution of marks obtained by 140 students in an examination:

| Marks | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | $90-99$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 7 | 15 | 18 | 25 | 30 | 20 | 16 | 7 | 2 |

Find the median of the distribution.

Ans: 51.167
7. Compute the median form the following data:

| Mid-value | 115 | 125 | 135 | 145 | 155 | 165 | 175 | 185 | 195 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| frequency | 6 | 25 | 48 | 72 | 116 | 60 | 38 | 22 | 3 |

Hints: The class intervals are: 110-120, 120-130,
-

Ans: Median = 153.79
8. You are given below a certain statistical distribution:

| Value | Less than 100 | $100-200$ | $200-300$ | $300-400$ | 400 and above | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 40 | 89 | 148 | 64 | 39 | 380 |

Calculate the most suitable average giving reasons for your choice.
Ans: Md. $=\mathbf{2 4 1 . 2 2}$
9. The following table gives the distribution of marks secured by some students in a certain examination:

| Marks | $:$ | $0-20$ | $21-30$ | $31-40$ | $41-50$ | $51-60$ | $61-70$ | $71-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students: | 42 | 38 | 120 | 84 | 48 | 36 | 31 |  |

Find (i) Median marks
(ii) The percentage of failure if minimum for a pass is 35 mraks

## Ans: (i) Md. $=\mathbf{4 0 . 4 6}$ (ii) $\mathbf{3 1 . 5 8 \%}$

10. Calculate the median from the following data:

Weight (in gms.) : 410-419 420-429 430-439 440-449 450-459 460-469 470-479 $\begin{array}{lllllllll}\text { No. of Apples } & : & 14 & 20 & 42 & 54 & 45 & 18 & 7\end{array}$ Ans: Median $=\mathbf{4 4 3 . 9 4}$ gms.
11. Given below is the distribution of 140 candidates obtaining marks X or higher in a certain examination (all marks are given in whole number):

| Marks (More than) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 140 | 133 | 118 | 100 | 75 | 45 | 25 | 9 | 2 | 0 |

Calculate the mean and median marks obtained by the candidates.
Ans: Mean = 50.714, Median $=51.167$
12. The following table gives the weekly wages in rupees in a certain commercial organisation:

| Weekly wages ('00 Rs) | $30-$ | $32-$ | $34-$ | $36-$ | $38-$ | $40-$ | $42-$ | $44-$ | $46-$ | $48-50$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3 | 8 | 24 | 31 | 5 | 61 | 38 | 21 | 12 | 2 |

Find: (i) the median and the first quartile, (ii) the number wage earners receiving between Rs 3,700 and Rs 4,700 per week.

$$
\text { Ans: (i) Md. = Rs } 4029.51, Q_{1}=\text { Rs } 3777.42 \text { (ii) } 191
$$

13. Define a percentile. Find the $45^{\text {th }}$ and $57^{\text {th }}$ percentiles for the following data on marks obtained by 100 students:

| Marks | $20-25$ | $25-30$ | $30-35$ | $35-40$ | $40-45$ | $45-50$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 10 | 20 | 20 | 15 | 15 | 20 |

Ans: $\mathbf{P}_{45}=33.75 ; \mathrm{P}_{57}=37.33$
14. Find:
(a) the $2^{\text {nd }}$ decile, (b) the $4^{\text {th }}$ decile (c) the $90^{\text {th }}$ percentile, and (d) the $68^{\text {th }}$ percentile for the data given below, interpreting clearly the significance of each:

| Age of Head of Family <br> (years) | Number <br> (in million) | Age of Head of Family <br> (years) | Number <br> (in million) |
| :---: | :---: | :---: | :---: |
| Under 25 | 2.22 | $55-64$ | 6.63 |
| $25-29$ | 4.05 | $65-74$ | 4.16 |


| 18 | 5.08 | 75 and over | $\underline{1.66}$ |
| :---: | :---: | :---: | :---: |
| $30-34$ | 10.45 |  | Total $\underline{43.72}$ |
| $35-44$ | 9.47 |  |  |

$$
\text { Ans: } D_{2}=31.94 \text { years } D_{4}=40.38 \text { years, } P_{90}=67.98 \text { years } P_{68}=52.87 \text { years }
$$

15. Find the (i) Lower quartile (ii) Upper quartile (iii) $7^{\text {th }}$ decile, and (iv) $60^{\text {th }}$ percentile, for the following frequency distribution:

| Wages (Rs) | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ | $80-90$ | $90-100$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Persons | 1 | 3 | 11 | 21 | 43 | 32 | 9 |

Ans: (i) Rs 67.14 (ii) Rs 83.44 (iii) Rs 81.56 (iv) Rs 78.37
16. 20. Draw an ogive for the data given below and show how can the value of median be read off from this graph. Verify you result.

| Class Interval : | $0-5$ | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency : | 5 | 10 | 15 | 8 | 7 | 5 |

Ans: Median = 13.5 (approx.); By formula, Md. $=13.33$
17. Draw a 'less than ogive' from the following data and hence find out the value of lower quartile:

| Class Interval : | $0-5$ | $5-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency $:$ | 5 | 10 | 15 | 20 | 8 | 5 |

Ans: $\mathbf{Q}_{\mathbf{1}}=12$
18. The frequency distribution of heights of 100 college students is as follows:

| Height (cms.) : | $141-150$ | $151-160$ | $161-170$ | $171-180$ | $181-190$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency $:$ | 5 | 16 | 56 | 19 | 4 | 100 |

Draw an ogive (less than or more than type) of this distribution and form the ogive find: (i) the first quartile (ii) the median (iii) the third quartile, and (iv) Inter-quartile range

Ans: $Q_{1}=161.2 \mathbf{c m s} . Q_{3}=170.1 \mathbf{c m s}$ Median $=165.7 \mathbf{c m s}$ I.Q. Range $=\mathbf{8 . 9} \mathbf{~ c m s .}$
19. The monthly salary distribution of 250 families in a certain locality in Agra is given below:

| Monthly Salary (Rs) | No. of Families | Monthly Salary (Rs) | No. of Families |
| :--- | :---: | :--- | :---: |
| More than 0 | 250 | More than 2,000 | 55 |
| More than 500 | 200 | More than 2,500 | 30 |
| More than 1,000 | 120 | More than 3,000 | 15 |
| More than 1,500 | 80 | More than 3,500 | 5 |

Draw a 'less than ogive' from the data given above and hence find out:
(i) Limits of the income of middle $50 \%$ of the families; and
(ii) If income-tax is to be levied on families whose income exceeds Rs 1,800 p.m.; calculate the percentage of families, which will be paying income-tax.
Ans: (i) Q1 = Rs 578 (approx); Q3 = Rs 1850
(ii) $\frac{25}{(2000-150)} \times(2000-1800)+25+15+10+5=65$

Percentage of families paying income tax $=\frac{65}{200} \times 100=\mathbf{2 6 \%}$
20. Draw a 'less than' and 'more than' ogive curve for the following data and find the median:

| No. of Children | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Families | 150 | 72 | 50 | 28 | 12 | 8 | 5 |

Hints: Since the number of children is a discrete random variable which take on positive integer values, the given frequency distribution can be expressed as grouped frequency distribution with exclusive type classes as given below:

| Frequency | 150 | 72 | 50 | 28 | 12 | 8 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ans; Median from ogive = 1.1 (aprox.) |  |  |  |  |  |  |  |
| 21. With th <br> (i) Valu <br> $\mathrm{D}_{6}$; (iv) | of given <br> iddle 50\% <br> hically wit | ata, find: of items; the help | (ii) Value of ogive cu | exactly 50 <br> ve, the val | $\begin{aligned} & \% \text { item; (ii } \\ & \text { es of } Q_{1}, Q \end{aligned}$ | The val median, | ue of $\mathrm{P}_{40}$ and $\mathrm{P}_{40}$ and $\mathrm{D}_{6}$ : |
| Class Interval | 10-14 | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | Total |
| Frequencies | 5 | 10 | 15 | 20 | 10 | 5 | 65 |
| Ans: (i) $26.75$ <br> 22. One hund distribu | $Q_{1}=29 .$ <br> and twe was obtai | $9-19.9$ <br> y studen d: | $2=9.27$ <br> ts appeare | $\text { Md. }=\mathbf{Q}$ <br> for a cert | $=25.13$ <br> in test an | ii) $\mathbf{P}_{40}=$ the follo | $23.17, D_{6}=$ <br> wing marks |
| Marks | 0 - |  | 20-40 | 40-60 | $60-$ |  | 80-100 |
| Students : | 10 |  | 30 | 36 | 30 |  | 14 |

Find: (i) The limits of marks of middle $30 \%$ of the students.
(ii) The percentage of students getting marks more than 75 .
(iii) The number of students who fail, if 35 marks are required for passing.

Ans: (i) $\mathrm{P}_{35}=41.1 ; \mathrm{P}_{65}=61.3$ (ii) $\frac{100}{120}\left[\left(\frac{30}{20} \times 5\right)+14\right]=17.9 \%$ (iii) $10+\frac{15}{20} \times 30=32.5$ $=33$
23. The expenditure of 1,000 families is given as under:

| Expenditure (in Rs) | $40-59$ | $60-79$ | $80-99$ | $100-119$ | $120-139$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No. of families | 50 | $?$ | 500 | $?$ | 50 |

The median for the distribution is Rs 87. Calculate the missing frequencies.
Ans: 262.5, 137.5 = 263, 137
24. An incomplete frequency distribution is given as follows:

| Variable | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 12 | 30 | $?$ | 65 | $?$ | 25 | 19 | 230 |

You are given that median value is 46 .
(a) Using the median formula, fill up the missing frequencies.
(b) Calculate the Arithmetic Mean of the completed table.

Ans: (a) 64, 45 (b) 45.96
25. An incomplete distribution is given below:

| Variable : | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency : | 10 | 20 | $?$ | 40 | $?$ | 25 | 15 |

(i) You are given that the median value is 35 . Find out missing frequency (given the total frequency $=170$ )
(ii) Calculate the arithmetic mean of the completed table.

Ans: (i) 35, 25 (ii) $\mathbf{3 5 . 8 8}$
26.

The data in the adjoining table represent travel expenses (other than transportation) for 7 trips made during November by a salesman for a small firm:

An auditor criticized these expenses as excessive, asserting that the average expense per day is Rs 10 (Rs 70 divided by 7). The salesman replied that the average is only Rs 4.20 (Rs 105 divided by 25) and that in any

| Trip | Days | Expenses <br> (Rs) | Expenses per <br> day (Rs) |
| :---: | :---: | :---: | :---: |
| 1 | 0.5 | 13.50 | 27 |
| 2 | 2.0 | 12.00 | 6 |
| 3 | 3.5 | 17.50 | 5 |
| 4 | 1.0 | 9.00 | 9 |
| 5 | 9.0 | 27.00 | 3 |

(8)


He presented these data to his father and asked for an increase in his allowance as he was getting less than average amount. His father, a statistician, countered pointing out that Atul's allowance was actually more than the average account.
Reconcile these statements.
Ans: Atul computed A.M. and his father computed Mode.
4. The number of fully formed apples on 100 plants were counted with following results:

| No. of apples | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of plants | 2 | 5 | 7 | 11 | 18 | 24 | 12 | 8 | 6 | 4 | 3 |

(i) How many apples were there in all?
(ii) What was the average of number of apples per plant?
(iii) What was the modal number of apples?

Ans: (i) 486 (ii) $\bar{X}=4.86$ (iii) Mo $=5$
5. Given below is the frequency distribution of marks obtained by 90 students. Compute the arithmetic mean, median and mode:

| Marks | No. of students | Marks | No. of students |
| :---: | :---: | :---: | :---: |
| $15-19$ | 6 | $45-49$ | 9 |
| $20-24$ | 14 | $50-54$ | 10 |
| $25-29$ | 12 | $55-59$ | 5 |
| $30-34$ | 10 | $60-64$ | 4 |
| $35-39$ | 10 | $65-69$ | 1 |
| $40-44$ | 9 |  |  |

Ans: Mean = 37.17; Md. = 36; mo. = 23.5
6. Find out the median and mode from the following table:

| No. of days absent | No. of students | No. of days absent | No. of students |
| :---: | :---: | :---: | :---: |
| Less than 5 | 29 | Less than 30 | 644 |
| Less than 10 | 224 | Less than 35 | 650 |
| Less than 15 | 465 | Less than 40 | 653 |
| Less than 20 | 582 | Less than 45 | 655 |
| Less than 25 | 634 |  |  |

Ans: Md. = 12.75; Mo. = 23.5
7. Find out the Mean, Median and the Mode in the following series:

| Size (below) | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 1 | 3 | 13 | 17 | 27 | 36 | 38 |

Ans: Md. = 12.75; Mo. = 11.35
8. In 500 small scale industrial units, the return on investment ranged from 0 to $30 \%$, no unit sustaining any loss. $5 \%$ of industrial units had returns exceeding $0 \%$ but not exceeding $5 \%, 15 \%$ of units had returns exceeding $5 \%$ but not exceeding $10 \%$. Median and upper quartiles ate of return was $15 \%$ and $20 \%$ respectively. The uppermost layer of return exceeding $25 \%$ but not exceeding $30 \%$ was earned by $25 \%$. Present the information in the form of frequency table with intervals as follows:

Exceeding 0\% but not exceeding 5\%
Exceeding 10\% but not exceeding 15\%
Exceeding 20\% but not exceeding 25\% Use N/4, 2N/4, 3N/4 as ranks of lower, middle and upper quartiles respectively. Find the rate of return around which there is maximum concentration of units.
Ans:

| Return in \% | $\mathbf{0 - 5}$ | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of units | 25 | 75 | 150 | 125 | 0 | 125 |

## 

Mode $=13.75$, Rate of return around which there is maximum concentration of units is $\mathbf{1 3 . 7 5 \%}$.
9. Calculate the arithmetic mean and the median of the frequency distribution given blow. Hence calculate the mode using the empirical relation between the three.


#### Abstract

Class limits: $\quad 130-134135-139140-144145-149150-154155-159160-164$


Frequency :
515
$\begin{array}{llll}28 & 24 & 17 & 10\end{array}$
$10 \quad 1$
Ans: $\mathrm{M}=145.35$; Md . $=144.92$; Mo. $=144.06$
10. (a) Briefly explain the role of grouping and analysis table in calculation of mode.
(b) From the following data of weight of 122 persons, determine the modal weight by the method of grouping:

| Weight (in lbs.) | $100-$ <br> 110 | $110-$ <br> 120 | $120-$ <br> 130 | $130-$ <br> 140 | $140-$ <br> 150 | $150-$ <br> 160 | $160-$ <br> 170 | $170-$ <br> 180 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons | 4 | 6 | 20 | 32 | 33 | 17 | 8 | 2 |

Hints: Method of grouping gives two modal classes 130-140 and 140-150 i.e. the distribution is bimodal. Locate the value of mode by using the empirical relation Mo $=3 M d-2 M$.

Ans: $\operatorname{Man}$ (M) = 139.51; Median (Md.) = 139.69; Mode (Md.) = 140.05
11. Calculate the Mode, Median and Arithmetic average from the following data:

| Class | $f$ | Class | $f$ |
| :---: | :---: | :---: | :---: |
| $0-2$ | 8 | $25-30$ | 45 |
| $2-4$ | 12 | $30-40$ | 60 |
| $4-10$ | 20 | $40-50$ | 20 |
| $10-15$ | 10 | $50-60$ | 13 |
| $15-20$ | 16 | $60-80$ | 15 |
| $20-25$ | 25 | $80-100$ | 4 |

Hints: Rewrite the frequency distribution with classes of equal magnitude 10.
Ans: Mo. = 28.15; Md. = 28.29; Mean = 30.08
12. In the following data, two class frequencies are missing:

| Class | Frequency | Class | Frequency |
| :---: | :---: | :---: | :---: |
| $100-110$ | 4 | $150-160$ | $?$ |
| $110-120$ | 7 | $160-170$ | 16 |
| $120-130$ | 15 | $170-180$ | 10 |
| $130-140$ | $?$ | $180-190$ | 6 |
| $140-150$ | 40 | $190-200$ | 3 |

However, it was possible to ascertain that the total number of frequencies was 150 and that the median has been correctly found to be 146.25
You are required to find out with the help of the information given:
(i) Two missing frequencies.
(ii) Having found the missing frequencies, calculate the arithmetic mean.
(iii) Without using the direct formula, find the value of mode.

Ans: (i) 24, 25 (ii) $\bar{X}=\mathbf{1 4 7 . 3 3}$ (iii) Mode $=144.08$
13. The median and mode of the following hourly wage distribution are known to be Rs 33.5 and Rs 34 respectively. Three frequency values form the table, however, missing. You are required to find out those values.

| Wages in Rs | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons | 4 | 16 | $?$ | $?$ | $?$ | 6 | 4 | 230 |

Ans: 60, 100, 40.
14. You are given the following incomplete frequency distribution. It is known that the total frequency is 1000 and that the median is 413.11 . Estimate by calculation the missing frequencies and find the value of the mode.

| Value (X) | Frequency (f) | Value (X) | Frequency (f) |
| :---: | :---: | :---: | :---: |
| $300-325$ | 5 | $400-425$ | 326 |
| $325-350$ | 17 | $425-450$ | $?$ |
| $350-375$ | 80 | $450-475$ | 88 |
| $375-400$ | $?$ | $475-500$ | 9 |

Ans: Missing frequencies are 227 and 248 respectively. Mo. $=413.98$
15. "Hari put the jar of water and the packet of sweets on the ground and sat down in the shade of the tree and waited".
Prepare a frequency distribution for the words in the above sentence taking the number of letters in words as the variable. Calculate the mean, median and mode.
Ans: Mean =3.56, Median = $\operatorname{Mode}=3$
16. Treating the number of letters in each word in the following passage as the variable $X$, prepare the frequency distribution table and obtain its mean, median, mode.
"The reliability of data must always be examined before any attempt is made to base conclusion upon them. This is true of all data, but particularly so of numerical data, which do not carry their quality written large on them. It is a waste of time to apply the refined theoretical methods of Statistics to data which are suspect from the beginning."
Ans: Mean = 4.565 Median = $4 \quad$ Mode $=3$
17. The frequency distribution of marks obtained by 60 students of a class in a college is given below:

| Marks | $:$ | $30-34$ | $35-39$ | $40-44$ | $45-49$ | $50-54$ | $55-59$ | $60-64$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students : | 3 | 5 | 12 | 18 | 14 | 6 | 2 |  |

(i) Draw a Histogram for this distribution and find the modal value.
(ii) Draw a cumulative frequency curve and find the marks limits of the middle $50 \%$ students.
Ans: (i) Mode $=\mathbf{4 7 . 5}$ marks (ii) $\mathbf{Q}_{\mathbf{1}}=\mathbf{4 2 . 5}$ marks $\mathbf{Q}_{\mathbf{3}}=\mathbf{5 2}$ marks
18. Determine the value of Median and Mode of the following distribution graphically. Verify the results by actual calculations. After verifying, calculate the value of Mean and sketch a curve indicating the general shape of the distribution and comment.

| Size | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | $90-99$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 11 | 19 | 21 | 16 | 10 | 8 | 6 | 3 | 1 |

Hint: Change classes into class boundaries for Md. and Mode. Use Ogive for Md. And Histogram for Mode graphically.
Use Formaul: Md. = 37.83, Mo. $=32.35$, Mean $=[(3 M d . ~-~ M o) / 2]=$.
$M>M d .>$ Mo. $\Rightarrow$ Distribution is positively skewed.
Exercise D (Geometric Mean and Harmonic Mean)

1. Compute the geometric mean of the following data:

| 35657 | 6754 | 431 | 32.5 | 4.17 | 0.897 | 0.05403 | 0.00678 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 16.14
2. Compute the geometric mean of the following data:

| 0.7894 | 0.0563 | 0.0041 | 0.6465 | 0.0003 | 0.0975 | 0.0854 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 0.04349
3. Compute the geometric mean of the following data:

| 100 | 150 | 360 | 200 | 500 | 450 | 750 | 800 | 900 | 5000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


4. From the following data, calculate the weighted geometric mean

| Head of Expenditure | Index Numbers | weight |
| :--- | :---: | :---: |
| Food | 150 | 6 |
| Shelter | 130 | 3 |
| Clothing | 120 | 4 |
| Education | 110 | 5 |
| Medicine | 90 | 2 |

Ans: 123.4
5. Compute the geometric mean of the following series:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 8 | 15 | 25 | 6 | 7 | 4 |

Ans: 21.03
6. Compute the Geometric Mean from the following distribution:

| Marks | $4-8$ | $8-12$ | $12-16$ | $16-20$ | $20-24$ | $24-28$ | $28-32$ | $32-36$ | $36-40$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 6 | 10 | 18 | 30 | 15 | 12 | 10 | 6 | 2 |

7. Compute the Geometric Mean from the following distribution:

| Marks | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 8 | 32 | 22 | 58 | 124 | 84 |

Ans: 36.5
8. Find the missing frequency of the following series, it its geometric mean is 17.15:

| Value | 10 | 15 | 25 | 12 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 3 | 5 | 2 | $?$ |

Ans: 2.03 or 2
9. Find the combined Geometric Mean of the following:

| Sections | A | B | C |
| :--- | :---: | :---: | :---: |
| No. of items | 8 | 7 | 5 |
| Geometric Mean | 8.52 | 10.12 | 7.75 |

Ans: 8.84
10. Determine the Geometric Mean of the following stock prices:

| Stock Price (in Rs) | $60-79.9$ | $80-99.9$ | $100-119.9$ | $120-139.9$ | $140-159.9$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 7 | 6 | 8 | 32 | 20 |

Ans: 121.31
11. Calculate the Harmonic Mean from the following series:

| $\mathrm{X}:$ | 1 | 10 | 100 | .1 | 0.1 | $0 . .1$ | 1.5 | 5.15 | 230 | .5 | .03 | .095 | .007 | 1275 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 0.0108
12. Compute the Weighted Harmonic Mean form the following data:

| Values | 1 | 0.4 | 1.5 | 45 | 174 | 0.01 | 3 | 12.5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weights | 8 | 15 | 2 | 15 | 10 | 20 | 10 | 5 |

Ans: 0.0414
13. Find the Harmonic Mean for the following data:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 7 | 11 | 16 | 17 | 25 | 30 |

Ans: 24.33
14. From the data given below, calculate the Harmonic Mean:

| Output | $10-14$ | $15-19$ | $20-24$ | $25-29$ | $30-34$ | $35-39$ | $40-44$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of workers | 2 | 6 | 7 | 12 | 15 | 5 | 3 |

Ans: 25.56
15. Find the missing frequency of the following series, if its harmonic mean is 20.08:

| Marks | 10 | 20 | 30 | 40 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- |



$$
\text { Ans: G.M. = 16.04; H.M. = } 1.7637
$$

20. Compute the geometric mean of the following data:

| 35657 | 6754 | 431 | 32.5 | 4.17 | 0.897 | 0.05403 | 0.00678 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 16.14
21. Compute the geometric mean of the following data:

| 0.7894 | 0.0563 | 0.0041 | 0.6465 | 0.0003 | 0.0975 | 0.0854 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 0.04349
22. Compute the geometric mean of the following data:

| 100 | 150 | 360 | 200 | 500 | 450 | 750 | 800 | 900 | 5000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Ans: 480.5
23. From the following data, calculate the weighted geometric mean

| Head of Expenditure | Index Numbers | weight |
| :--- | :---: | :---: |
| Food | 150 | 6 |
| Shelter | 130 | 3 |
| Clothing | 120 | 4 |
| Education | 110 | 5 |
| Medicine | 90 | 2 |

Ans: 123.4
24. Compute the geometric mean of the following series:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 8 | 15 | 25 | 6 | 7 | 4 |

Ans: 21.03
25. Compute the Geometric Mean from the following distribution:

| Marks | $4-8$ | $8-12$ | $12-16$ | $16-20$ | $20-24$ | $24-28$ | $28-32$ | $32-36$ | $36-40$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 6 | 10 | 18 | 30 | 15 | 12 | 10 | 6 | 2 | Ans:

26. Compute the Geometric Mean from the following distribution:

| Marks | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 8 | 32 | 22 | 58 | 124 | 84 |




| S. No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income in Rs | 145 | 367 | 268 | 73 | 185 | 619 | 280 | 115 | 870 | 315 |

Ans: (a) Md = Rs 274 (b) G = Rs 252.4 (c) Coefficient of Range $=0.84$
2. The index number of prices of cotton shares $\left(I_{1}\right)$ and coal shares $\left(I_{2}\right)$ in a given year are as under:

| Months: | Jan. | Feb. | Mar. | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{I}_{1}$ | 188 | 178 | 173 | 164 | 172 | 183 | 184 | 185 | 211 | 217 | 232 | 240 |
| $\mathrm{I}_{2}$ | 131 | 130 | 130 | 129 | 129 | 120 | 127 | 127 | 130 | 137 | 140 | 142 |

Calculate range for each share. Hence, discuss which share do you consider more variable in price.
Ans: Range $\left(I_{1}\right)=76$, Coefficient of Range $\left(I_{1}\right)=0.19$; Range $\left(I_{2}\right)=22$, Coefficient of Range ( $I_{2}$ ) = 0.084; Hence cotton shares are more variable in prices.
3. Age distribution of 200 employees of a firm is given below: Construct a less than ogive curve and hence or otherwise calculate semi-inter quartile rage $\frac{Q_{3}-Q_{1}}{2}$ of the distribution:

| Age in years (less than) | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of employees | 10 | 25 | 75 | 130 | 170 | 189 | 200 |

Ans: $\mathbf{Q}_{1}=33.5$ years $\quad Q_{3}=43$ years $\quad \frac{Q_{3}-Q_{1}}{2}=4.75$ years
4. Find the mode, median, lower quartile $\left(Q_{1}\right)$ and upper quartile $\left(Q_{3}\right)$ and Coeff. of $Q$. D. from the following data:

| Wages: | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No. of workers: | 22 | 38 | 46 | 35 | 20 |

Ans: Mode $=24.21$; Median $=24.46$ Q $_{1}=14.803 Q_{3}=24.21$ Coeff. of Q. D. $=0.396$
5. Calculate the Coefficient of Quartile Deviation of the following data:



Ans: Q.D. = 1.5; M.D. (about median) $=1.73$
4. With median as base, calculate the mean deviation and compare the variability of two series A and B.

| Series A: | 3484 | 4572 | 4124 | 3682 | 5624 | 4388 | 3680 | 4308 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Series B: | 487 | 508 | 620 | 382 | 408 | 266 | 186 | 218 |

Ans: $\quad$ Series A: Md. = 4216 ; M.D. = 490.25; Coefficient of M.D. $=0.116$
Series A: Md. = 395 ; M.D. $=121.38$; Coefficient of M.D. $=0.307$.
Series B is more variable.
5. Compare the dispersion of the following series by using the co-efficient of mean deviation.

| Age (years) : | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of boys : | 4 | 5 | 7 | 12 | 20 | 13 | 5 | 0 | 4 | 70 |
| No. of girls : | 2 | 0 | 4 | 8 | 15 | 10 | 6 | 3 | 2 | 50 |

Ans: Coefficient of M.D. about median (boys) $\mathbf{= 0 . 0 6 8 5}$; Coefficient of M.D. about median (girls) $=0.0630$
6. Calculate the mean deviation from mean for the following data:

| Marks: | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 6 | 5 | 8 | 15 | 7 | 6 | 3 |

Ans: Mean = 33.4; M.D. about mean = 13.184
[C.A. Foundation, May 1999]
7. Find out mean deviation and its coefficient from median from the following series:

| Size of items: | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency: | 2 | 1 | 3 | 6 | 4 | 3 | 1 |

Ans: 2.4; 0.24
8. Calculate the mean deviation about the mean for the following data:

| X | 5 | 15 | 25 | 35 | 45 | 55 | 65 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | 8 | 12 | 10 | 8 | 3 | 2 | 7 |

Also find the M.D. about median and comment on the results obtained in (a) and (b)
Ans: Mean = 29; M.D. about mean = 16; Median = 22; M.D. about median =15.8
9. Calculate mean deviation from median from the following data:

17. Find the mean and standard deviation of the following data:

| Age under (years) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons dying | 15 | 30 | 53 | 7 | 100 | 110 | 115 | 125 |

Ans: Mean = 35.16; S.D. $=19.76$ years
18. In the following data, two class frequencies are missing:

| Class Interval | Frequency | Class Interval | Frequency |
| :---: | :---: | :---: | :---: |
| $100-110$ | 4 | $150-160$ | $?$ |
| $110-120$ | 7 | $160-170$ | 16 |
| $120-130$ | 15 | $170-180$ | 10 |
| $130-140$ | $?$ | $180-190$ | 6 |
| $140-150$ | 40 | $190-200$ | 3 |

However, it was possible to ascertain that the total number of frequencies was 150 and that the median has been correctly found out as 146.25 . you are required to find with the help of information given:
(i) The two missing frequencies;
(ii) Having found the missing frequencies, calculate the Arithmetic Mean and Standard Deviation
(iii) Without using the direct formula, find the value of mode.

Ans: (i) 24, 25 (ii) A.M. = 147.33; S.D. = 19.2 (iii) Mode $=144.09$
19. The following table gives the distribution of income of household based on hypothetical data:

| Income <br> $($ Rs $)$ | Percentage of <br> households | Income <br> $(\mathrm{Rs})$ | Percentage of <br> households |
| :---: | :---: | :---: | :---: |
| Under 100 | 7.2 | $500-599$ | 14.9 |
| $100-199$ | 11.7 | $600-699$ | 10.4 |
| $200-299$ | 12.1 | $700-999$ | 9.0 |
| $300-399$ | 14.8 | 1,000 and above | 4.0 |
| $400-499$ | 15.9 |  |  |

(i) What are the problems involved computing standard deviation from the above data?
(ii) Compute a suitable measure of dispersion

Ans: (ii) Compute Quartile Deviation; Q. D. = 169.425; Co-efficient of Q.D. = 0.404 EXERCISE C (Skewness)

1. From the following distribution of marks, find the Karl Pearson's skewness and its coefficient:

| Roll No. | 1001 | 1002 | 1003 | 1004 | 1005 | 1006 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Marks | 4 | 6 | 20 | 10 | 7 | 3 |

Ans: SK(p) = 5.49; Coefficient of SK(p) = 0.97
2. From the following frequency distribution, find out Karl Pearson's skewness and its coefficient:

| Wages in Rs | 10 | 20 | 30 | 40 | 50 | 60 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of workers | 3 | 7 | 10 | 20 | 6 | 4 |

Ans: SK(p) = - 3.8; Coefficient of SK(p) = - 0.30
3. From the following distribution, find out Karl Pearson's coefficient of skewness:

| Group | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ | $30-35$ | $35-40$ | $40-45$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F: | 5 | 10 | 12 | 14 | 30 | 18 | 9 | 2 |

Ans: SK(p) = - 0.32
4. From the following data, find Bowley's skewness and its coefficient

| Size | 1 | 5 | 12 | 22 | 17 | 9 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


4. Calculate Karl Pearson's coefficient of correlation from the following data, using 20 as the working mean for price and 70 as the working mean for demand:

| Price | 14 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Demand | 84 | 78 | 70 | 75 | 66 | 67 | 62 | 58 | 60 |

5. Ans: $\mathbf{r}=-\mathbf{0 . 9 5 4}$
6. Calculate the Karl Pearson's coefficient of correlation from the following data:

|  |  | Percentage of Marks |  |  |  |  | Percentage of Marks |  |
| :--- | :--- | :---: | :---: | :--- | :--- | :--- | :--- | :---: |
| No. | Subject | First Term | Second Term | No. | Subject |  | First Term | Second Term |
| 1. | Hindi | 75 | 62 | 5. | Commerce | 77 | 69 |  |
| 2. | English | 81 | 68 | 6. | Mathematics | 81 | 72 |  |
| 3. | Economics | 70 | 65 | 7. | Statistics | 84 | 76 |  |
| 4. | Accounts | 76 | 60 | 8. | Costing | 75 | 72 |  |

Ans: $\mathbf{r}=\mathbf{0 . 6 2 3}$
7. Calculate the Karl Pearson's coefficient of correlation for the following ages of husbands and wives at the time of their marriage:

| Age of husband (in years) | 23 | 27 | 28 | 28 | 28 | 30 | 30 | 33 | 35 | 38 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Age of wife (in years) | 18 | 20 | 22 | 27 | 21 | 29 | 27 | 29 | 28 | 28 |

Ans: $\mathbf{r}=\mathbf{0 . 8 0 1 3}$
8. Calculate Pearson's coefficient of correlation from the following data using 44 and 26 respectively as the origin of X and Y :

| X: | 43 | 44 | 46 | 40 | 44 | 42 | 45 | 42 | 38 | 40 | 42 | 57 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y: | 29 | 31 | 19 | 18 | 19 | 27 | 27 | 29 | 41 | 30 | 26 | 10 |

Ans: $\mathrm{r}_{\mathrm{xy}}=-\mathbf{0 . 7 3 2 6}$
9. The following table gives the distribution of total population and those who are totally or partially blind among them. Find out if there is any relation between age and blindness:

| Age | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons <br> $(‘ 000)$ | 100 | 60 | 40 | 36 | 24 | 11 | 6 | 3 |
| Blind | 55 | 40 | 40 | 40 | 36 | 22 | 18 | 15 |

Hints: Here we shall find the correlation coefficient between age $(X)$ and no. of blinds per lakh $(Y)$ as given in the following table

| X | 5 | 15 | 25 | 35 | 45 | 55 | 65 | 75 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 55 | 67 | 100 | 111 | 150 | 200 | 300 | 500 |

Ans: $\mathbf{r}=\mathbf{0 . 8 9 8 2}$
10. With the following data in 6 cities, calculate the coefficient of correlation by Pearson's method between the density of population and the death rate:

| Cities | Area in square miles | Population (in '000) | No. of deaths |
| :--- | :---: | :---: | :---: |
| A | 150 | 30 | 300 |
| B | 180 | 90 | 1440 |
| C | 100 | 40 | 560 |
| D | 60 | 42 | 840 |
| E | 120 | 72 | 1224 |
| F | 80 | 24 | 312 |

Hint: Find r between, Density $=\frac{\text { Population }}{\text { Area }} ;$ and Death Rate $=\frac{\text { No.of Deaths }}{\text { Population }} \times 1000$
Ans: $\mathbf{r}=\mathbf{0 . 9 8 7 6}$
11. Calculate the correlation coefficient from the following data:
12. Let now each value of $X$ be multiplied by 2 and then 6 be added to it. Similarly multiply each value of Y by 3 and subtract 2 from it. What will be the correlation coefficient between the new series of X and Y ?

## [C. A. Foundation, May 1997]

Ans: $\mathbf{r}(\mathbf{X}, \mathrm{Y})=\mathbf{0 . 9 4 8 5}$
13. 18. Given: $\sum X=125, \sum Y=100, \sum X^{2}=650, \sum Y^{2}=436, \sum X Y=520$ and $\mathrm{n}=25$, obtain the value of Karl Pearson's correlation coefficient r (X,Y)

Ans: 0.67
14. You are given the following information relating to a frequency distribution comprising of 10 observations:
$\bar{X}=5.5, \bar{Y}=4.0, \sum X^{2}=385, \sum Y^{2}=192, \sum(X+Y)^{2}=947$
Find $\mathrm{r}_{\mathrm{xy}}$
Ans: $\mathbf{r}(X, Y)=0.681$
15. A computer while calculating the correlation coefficient between the variables $X$ and $Y$ obtained the following results:
$\mathrm{n}=30, \sum X=120, \sum X^{2}=600 \sum Y=90,, \sum Y^{2}=250, \sum X Y=356$
It was, however, later discovered at the time of checking that it had copied down two pairs of observations as:

| X | Y |
| :---: | :---: |
| 8 | 10 |
| 12 | 7 |

While the correct values were,

| X | Y |
| :---: | :---: |
| 8 | 12 |
| 10 | 8 |

Obtain the correct value of the correlation coefficient between X and Y .
Ans: $\mathbf{r}=\mathbf{0 . 0 5 0 4}$
[I.C.W.A. December, 2003]
16. Coefficient of correlation between $X$ and $Y$ for 20 items is 0.3 ; mean of $X$ is 15 and that of $Y$ is 20, standard deviations are 4 and 5 respectively. At the time of calculations one pair ( $\mathrm{X}=$ $27, \mathrm{Y}=30$ ) was wrongly taken as $(\mathrm{X}=17, \mathrm{Y}=35)$. Find the correct coefficient of correlation.

Ans: Correct value of correlation coefficient $=\mathbf{0 . 5 1 5 3}$
17. In order to find the correlation coefficient between variables $X$ and $Y$ from 12 pairs of observations, the following calculations were made: $\sum X=30, \sum Y=5, \sum X^{2}=670, \sum Y^{2}=285, \sum X Y=334$
On subsequent verification, it was found that the pair $(X=11, Y=4)$ was copied wrongly, the correct value being $(X=10, Y=14)$. Find the correct value of correlation coefficient.

Ans: 0.78
18. Calculate the coefficient of correlation and find its probable error from the following data:

| X: | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 18 | 16 | 14 | 12 | 10 | 6 | 8 |

Ans: $\mathbf{r x y}=\mathbf{0 . 9 6 4 3}$; P.E. $(\mathbf{r})=0.0179$
19. Find Karl Pearson's correlation coefficient between age and playing habits of the following students:

| Age (Years) | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 250 | 200 | 150 | 120 | 100 | 80 |
| No. of Regular players | 200 | 150 | 90 | 48 | 30 | 12 |

## Hint: Find $r$ between age $(X)$ and percentage of regular players $(Y)$.

Ans: $\mathbf{r}_{\mathrm{xy}}=\mathbf{- 0 . 9 9 1 2}$
20. Calculate Karl Pearson's coefficient of correlation for the following series:

| Price (in Rs) | $110-111$ | $111-112$ | $112-113$ | $113-114$ | $114-115$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Demand (in Kg.) | 600 | 640 | 640 | 680 | 700 |
| Price (in Rs) | $116-117$ | $117-118$ | $118-119$ |  |  |
| Demand (in Kg.) | 830 | 900 | 1,000 |  |  |

Also calculate the probable error of the correlation coefficient. From you result can you assert that the demand is correlated with price?

$$
\text { Ans: } \mathrm{r}=0.9651 ; \text { P.E. }(\mathrm{r})=0.0154
$$

21. The deviations from the respective means of $X$ and $Y$ series are given below:

| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 3 | -3 | -4 | 0 | 4 | 1 | 2 | -2 | -1 |

Calculate Karl Pearson's coefficient of correlation from the above data.
Ans: $\mathbf{r}_{(\mathrm{X}, \mathrm{Y})}=0$
22. Calculate the coefficient of correlation between $X$ and $Y$ series from the following data:

|  | X Series | Y Series |
| :--- | :---: | :---: |
| No. of observations | 15 | 15 |
| Arithmetic mean | 25 | 18 |
| Standard deviation | 5 | 5 |

$\sum(X-25)(Y-18)=125$
Ans: $\mathbf{r}_{\mathrm{xy}}=\mathbf{0 . 3 3}$
23. Given $\mathrm{n}=10, \sum X=100, \sum(X-10)^{2}=600 \sum Y=150,, \sum(Y-15)^{2}=25$, $\sum(X-10)(Y-15)=60$

Ans: $r=2 \sqrt{5}=0.8944$
24. The following results are obtained between two series. Compute the coefficient of correlation:

|  | X Series | Y Series |
| :--- | :---: | :---: |
| Number of items | 7 | 7 |
| Arithmetic mean | 4 | 8 |
| Sum of squares of deviations from arithmetic mean | 28 | 76 |
| Summation of products of deviations of X and Y series from their respective means = 46 |  |  |

Ans: 0.997

## Exercise - B (Rank Correlation)

1. Ranking of 10 trainees at the beginning $(X)$ and at the end $(Y)$ of a certain course are given below:

| Trainees | A | B | C | D | E | F | G | H | I | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X: | 1 | 6 | 3 | 9 | 5 | 2 | 7 | 10 | 8 | 4 |
| Y: | 6 | 8 | 3 | 7 | 2 | 1 | 5 | 9 | 4 | 10 |

Calculate Spearman's rank correlation coefficient. [I.C.W.A. Inter, June 1995]
Ans: $\mathbf{R}=0.394$
2. The rank of same 16 students in Mathematics and Physics are as follows. Two number within brackets denote the ranks of the students in Mathematics and Physics.
$(1,1)(2,10)$
$(3,3)(4,4)$
$(5,5)(6,7)(7,2)$
) $(8,6$
6) $(9,8)$
$(10,11$

1) $(11,15$
$(12,9)$
$(13,14$
(14,
2) $(15,16)(16,13)$

Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Physics.

Ans: $\mathrm{R}=0.8$
3. Two judges in a beauty competition rank the 12 entries as follows:

| $\mathrm{X}:$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}:$ | 12 | 9 | 6 | 10 | 3 | 5 | 4 | 7 | 8 | 2 | 11 | 1 |

What degree of agreement is there between the two judges.
Ans: $\mathrm{R}=\mathbf{- 0 . 4 5 4}$
4. Ten competitors in a beauty contest are ranked by three judges in the following order:

| 1st $^{\text {st }}$ Judge | 1 | 5 | 4 | 8 | 9 | 6 | 10 | 7 | 3 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| 2 $^{\text {d }}$ Judge | 4 | 8 | 7 | 6 | 5 | 9 | 10 | 3 | 2 | 1 |
| 3 $^{\text {rd }}$ Judge | 6 | 7 | 8 | 1 | 5 | 10 | 9 | 2 | 3 | 4 |

Use rank correlation coefficient to discuss which pair of judges has the nearest approach to beauty.

$$
\text { Ans: } \mathrm{R}_{12}=0.5515, \mathrm{R}_{13}=0.0545, \mathrm{R}_{23}=0.7333
$$

The pair of $2^{\text {nd }}$ and $3^{\text {rd }}$ judges has the nearest approach to common tastes in beauty.
5. For the following data, calculate the Coefficient of Rank Correlation:

| $\mathrm{X}:$ | 80 | 91 | 99 | 71 | 61 | 81 | 70 | 59 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}:$ | 123 | 135 | 154 | 110 | 105 | 134 | 121 | 106 |
|  |  |  |  |  |  |  |  | $\mathrm{R}=0.9524$ |

6. The following are the marks obtained by a group of students in two papers. Calculate the rank coefficient of correlation:

| Economics: | 78 | 36 | 98 | 25 | 75 | 82 | 92 | 62 | 65 | 39 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Statistics: | 84 | 51 | 91 | 69 | 68 | 62 | 86 | 58 | 35 | 49 |

7. Calculate Spearman's coefficient of rank correlation for the following data of scores in psychology (X) and arithmetical ability (Y) of 10 children:

| Child | A | B | C | D | E | F | G | H | I | J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X: | 105 | 104 | 102 | 101 | 100 | 99 | 98 | 96 | 93 | 92 |
| Y: | 101 | 103 | 100 | 98 | 95 | 96 | 104 | 92 | 97 | 94 |
|  |  |  |  |  |  |  |  |  |  | R = 0.6 |

8. Compute the Coefficient of Rank Correlation between $X$ and $Y$ from the data given below:

$$
\begin{array}{lccccccccccccccc}
\mathrm{X}: & 8 & 10 & 7 & 15 & 3 & 20 & 21 & 5 & 10 & 14 & 8 & 16 & 22 & 19 & 6 \\
\mathrm{Y}: & 3 & 12 & 8 & 13 & 20 & 9 & 14 & 11 & 4 & 16 & 15 & 10 & 18 & 23 & 25
\end{array}
$$

Ans: $\mathrm{R}=0.0357$
9. Given the following aptitude and I. Q. scores for a group of students. Find the coefficient of rank correlation:

| Aptitude Score | 57 | 58 | 59 | 59 | 60 | 61 | 60 | 64 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I.Q. Score | 97 | 108 | 95 | 106 | 120 | 126 | 113 | 110 |

Ans: $\mathrm{R}=0.7024$
10. The following data relate to the marks obtained by 10 students of a class in Statistics and Costing:
$\begin{array}{llllllllllll}\text { Marks in Statistics } & 30 & 38 & 28 & 27 & 28 & 23 & 30 & 33 & 28 & 35\end{array}$
$\begin{array}{llllllllllll}\text { Marks in Costing } & 29 & 27 & 22 & 29 & 20 & 29 & 18 & 21 & 27 & 22\end{array}$
Obtain the rank correlation coefficient.
Ans: $\mathrm{R}=0.3515$
11. Find the coefficient of rank correlation between the marks obtained in Mathematics (X) and those in Statistics (Y) by 10 students of a certain class out of a total marks of 50 in each subject:
$\begin{array}{llllllllllll}\text { Student No.: } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10\end{array}$

| $\mathrm{X}:$ | 12 | 18 | 32 | 18 | 25 | 24 | 25 | 40 | 38 | 22 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 16 | 15 | 28 | 16 | 24 | 22 | 28 | 36 | 34 | 19 |

Ans: $\mathrm{R}=0.95$
12. From the following data, calculate the coefficient of rank correlation between $X$ and $Y$ :

| X: | 32 | 35 | 49 | 60 | 43 | 37 | 43 | 49 | 10 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y: | 40 | 30 | 70 | 20 | 30 | 50 | 72 | 60 | 45 | 25 |

Ans: $\mathrm{R}=\mathbf{-} 0.0758$

## EXERCISE C (REGRESSION ANALYSIS )

1. Given the following values of X and Y :

| $\mathrm{X}:$ | 3 | 5 | 6 | 8 | 9 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}:$ | 2 | 3 | 4 | 6 | 5 | 8 |

Find the equation of regression of:
(i) Y on X and (ii) X on Y
2. Obtain the equations of the two lines of regression for the data given below:

| $\mathrm{X}:$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}:$ | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 16 | 15 |

Ans: $Y=0.95 X+7.25 ; X=0.95 Y+7.25$
3. From the following data of the age of husband and the age of wife, form two regression lines and calculate the husband's age when the wife's age is 16 :

| Husband's age: | 36 | 23 | 27 | 28 | 28 | 29 | 30 | 31 | 33 | 35 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Wife's age: | 29 | 18 | 20 | 22 | 27 | 21 | 29 | 27 | 29 | 28 |

Ans: Husband's age: X : Wife's age: Y
$\mathbf{Y}=\mathbf{0 . 9 5 X} \mathbf{- 3 . 5}, \quad \mathrm{X}=\mathbf{0 . 8} \mathrm{Y}+\mathbf{1 0 ( X ) _ { Y = 1 6 } = 2 2 . 8}$
4. Find the regression equation Y on X where Y and x are the marks obtained by 10 students as given below:

| $\mathrm{Y}:$ | 20 | 60 | 55 | 45 | 75 | 35 | 25 | 90 | 10 | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}:$ | 20 | 45 | 65 | 40 | 55 | 35 | 15 | 80 | 25 | 50 |

Ans: $b_{\mathrm{xy}}=1.105 ; Y=1.105 \mathrm{X}-1.015 \quad$ [C.A. (Foundation) May, 2002]
5. The following data give the experience of machine operators and their performance ratings as given by number of good parts turned out per pieces:

| Operator | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Experience (in years) (X) | 16 | 12 | 18 | 4 | 3 | 10 | 5 | 12 |
| Performance Ratings (Y) | 87 | 88 | 89 | 68 | 78 | 80 | 75 | 83 |

Calculate the regression line of performance ratings on experience and estimate the probable performance if an operator has 7 years' experience.
Ans: $\mathbf{Y}=\mathbf{6 9 . 9 7}+\mathbf{1 . 1 3 3 X} ; 77.601$
6. You are given the data relating to purchases and sales. Obtain the two regression equations by the method of least squares and estimate the likely sales when the purchases equal 100:

| Purchases: | 62 | 72 | 98 | 76 | 81 | 56 | 76 | 92 | 88 | 49 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales: | 112 | 124 | 131 | 117 | 132 | 96 | 120 | 136 | 97 | 85 |

Ans: Purchases X; Sales Y; X = 0.6515Y + 0.0775; Y = 0.7825X + 56.3125; 134.5625
7. The height of fathers and sons is given in the following table. Find the two lines of regression and estimate the expected average height of the son when the height of the father is 67.5 inches:

| Height of father (in inches) | 65 | 66 | 67 | 67 | 68 | 69 | 71 | 73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Height of sons (in inches) | 67 | 68 | 64 | 68 | 72 | 70 | 69 | 70 |

Ans: $Y=0.4242 X+39.5484 ; X=0.525 Y+32.2875 ; 68.18$ inches
8. The following table gives the ages and blood pressure of 10 women:

| Age (Y) | 56 | 42 | 36 | 47 | 49 | 42 | 60 | 72 | 63 | 55 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blood Pressure (Y) | 147 | 125 | 118 | 128 | 145 | 140 | 155 | 160 | 149 | 150 |

(i) Find the correlation coefficient between X and Y .
(ii) Determine the least square regression equation Y on X .
(iii) Estimate the blood pressure of a woman whose age is 45 years.

$$
\text { Ans: (i) } r=0.89 \text { (ii) } Y=83.758+1.11 X \text { (iii) } W \text { hen } X=45, Y=134
$$

9. A panel of two judges P and Q graded seven dramatic performances independently by
awarding marks as follows:

| Performance: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Marks of P: | 46 | 42 | 44 | 40 | 43 | 41 | 45 |
| Marks of Q: | 40 | 38 | 36 | 35 | 39 | 37 | 41 |

The eighth performance, which Judge Q could not attend, was awarded 37 marks by Judge P. If Judge Q has also been present, how many marks would be expected to have been awarded by him to the eighth performance?

Ans: $33.5=34$

10 .
10. The following table gives the normal weight of baby during the first six months of life:

| Age in months | 0 | 2 | 3 | 5 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Weight in lbs. | 5 | 7 | 8 | 10 | 12 |

Estimate the weight of a baby at the age of 4 months.

## Ans: 9.2982 lbs

11. You are given the following data:

|  | X | Y |
| :--- | :---: | :---: |
| Arithmetic Mean | 36 | 85 |
| Standard Deviation | 11 | 8 |

Correlation coefficient between X and $\mathrm{Y}=0.66$
(i) Find two regression equations
(ii) Estimate the value of X when $\mathrm{Y}=75$

$$
\text { Ans: (i) } Y=0.48 X+67.72 ; X-0.9075 Y-41.1375 ; \text { (ii) } 26.925
$$

12. 34. A survey was conducted to study the relationship between expenditure on accommodation $(\mathrm{X})$ and expenditure on food and entertainment $(\mathrm{Y})$ and the following results were obtained:

|  | Mean | S. D. |
| :--- | :---: | :---: |
| Expenditure on accommodation | Rs 173 | 63.15 |
| Expenditure on food and entertainment | Rs 47.8 | 22.98 |

Coefficient of correlation $=+0.57$
Write down the equation of regression of X on Y and estimate the expenditure on food and entertainment, if the expenditure on accommodation is Rs 200.

Ans: $Y=0.207 \mathrm{X}+11.99, Y_{X=200}=$ Rs 53.29
13. Find out the regression coefficients $Y$ on $X$ and $X$ on $Y$ on the basis of the following data: $\sum X=50, \bar{X}=5, \sum Y=60, \bar{Y}=6, \sum X Y=350$, Variance of $X=4$, Variance of $Y=9$

Ans: $b_{y x}=1.25, b_{x y}=0.56$
14. In order to find the correlation coefficient between two variables $X$ and $Y$ from 12 pairs of observations, the following calculations were made:

$$
\sum X=30, \sum X^{2}=670, \sum Y=5, \sum Y^{2}=285, \sum X Y=344
$$

On subsequent verification, it was discovered that the pair $(X=11, Y=4)$ was copied wrongly, the correct values being $(X=10, Y=14)$. After making necessary correction, find:
(a) the two regression coefficients; (b) the two regression equations;
(c) the correlation coefficient.

Ans: (a) $b_{y x}=0.694 ; b_{x y}=0.898$ (b) : Yon $X: Y=0.694 X-0.427$ : $X$ on $Y: X=0.898 Y+$ $1.294(\mathrm{c}) \mathrm{r}_{(\mathrm{x}, \mathrm{y})}=0.7894=0.79$

## UNIT - IV

1. Define an index number. Discuss the essential characteristics of an index number.
2. What are the index numbers? Discuss the different problems that we faced in the construction of an index number.
3. Discuss the importance and limitations of index numbers.
4. What do you mean by cost of living index numbers? Discuss the methods for construction of such index numbers.
5. What is a fixed based index number? Differentiate it from chain base index numbers. Discuss the merits and demerits of both the index numbers.
6. What is Fisher's ideal index number? What is it so called? Show that it satisfies to both the time reversal test and factor reversal test.
7. 'Index numbers are called the economic barometer of a country'. Comment on the statement and also bring out the limitations of the index numbers.
8. What are different methods of measuring trend? Explain the methods of eliminating trend in a time series. Which one do you consider better?
9. Explain briefly the various methods of determining the trend in a time series.
10. Explain 'ratio to trend' method of measuring seasonal variations and discuss its relative merits and demerits.

## EXERCISE A (General Price Index)

1. On the basis of figures of production of generators given below, construct:
(a) Quantity index; and (b) Price index (using 1990 as base)

| Year | 1990 | 1991 | 1992 | 1993 | 1994 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Units Produced (in thousands) | 24 | 30 | 32 | 38 | 44 |
| Value of Output (in Rs Million) | 192 | 255 | 272 | 361 | 451 |

Ans:

| Year | 1990 | 1991 | 1992 | 1993 | 1994 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price Index | 100 | 106.25 | 106.25 | 118.75 | 128.12 |
| Quantity Index | 100 | 125 | 133.33 | 158.33 | 183.33 |

2. What is the difference between Laspeyre's and Paasche's system of weights in compiling a price index? Calculate both Laspeyre's and Paasche's aggregative price indices for the year 2000 from the following data:

| Commodities | Quantity |  | Price Per Unit (Rs) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1999 | 2000 | 1999 | 2000 |
| A | 3 | 5 | 20 | 25 |
| B | 4 | 6 | 25 | 30 |
| C | 2 | 3 | 30 | 25 |
| D | 1 | 2 | 10 | 7.50 |

Ans: 109.78; 109.72
3. From the data given below compute Laspeyr's and Paasche's index numbers:

| Commodities | Price |  | Quantity |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1995 | 2001 | 1995 | 2001 |
| A | 4 | 10 | 50 | 40 |
| B | 3 | 9 | 10 | 2 |
| C | 2 | 4 | 5 | 2 |

(Price and Quantity figures are in appropriate units)
Ans: 254.16; 250.58
4. The geometric mean of index number of Laspeyre and Paasche is 229.5648 while the sum of Laspeyre's and Paasche's index number is 480. Find out Laspeyre's and Paasche's indices.

Ans: 310; 170
5. Using Paasche's formula, compute the quantity index and the price index number of 2000 with 1996 as base year:

| Commodities | Quantity Units |  | Value in (Rs) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1999 | 2000 | 1999 | 2000 |
| A | 100 | 150 | 500 | 900 |
| B | 80 | 100 | 320 | 500 |
| C | 60 | 72 | 150 | 360 |
| D | 30 | 33 | 360 | 297 |

(b) For the above problem also compute price index by
(i) Marshall Edgeworth formula; (ii) Fisher's formula (iii) Dorbish-Bowley formula; (iv) Walsch formula
Ans: (a) $\mathrm{P}_{\mathbf{0 1}}=119.2 ; \mathrm{Q}_{01}=\mathbf{1 3 1 . 0 9}$ (b) (i) 118.68 (ii) 118.62 (iii) 118.6225 (iv) 118.64

| Year | Rice |  | Wheat |  | Jowar |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Price | Quantity | Price | Quantity | Price | Quantity |
| 1970 | 9.3 | 100 | 6.4 | 11 | 5.1 | 5 |
| 1977 | 4.5 | 90 | 3.7 | 10 | 2.7 | 3 |

Ans: 49.135: 49.134
7. A company spent Rs 50 , Rs 48 , Rs 18 and Rs 42 during 1998. The company increased the expenditure to Rs 100 , Rs 98 , Rs 60 and Rs 102 respectively on four commodities. If the units of four commodities purchased during 1998 and 1999 are identical i.e. 5, 2, 6 and 17, compute the price index for 1999 by the most suitable method.

Ans: Simple Aggregative method 227.85
8. From the data given below construct an index number of the group of four commodities using:
(i) Simple Aggregative Method and (ii) Fisher's Ideal Formula.

| Commodities | Base Year (1996) |  | Current Year (1997) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price per unit | Expenditure <br> (Rs) | Price per unit | Expenditure (Rs) |
| 1 | 2 | 40 | 5 | 75 |
| 2 | 4 | 16 | 8 | 40 |
| 3 | 1 | 10 | 2 | 24 |
| 4 | 5 | 25 | 10 | 60 |

Ans: (i) 208.33 (ii) 219.13
9. 1Using Fisher's Ideal Formula, compute price and quantity index numbers for 11984 with 1982 as base year, given the following information:

| Year | Commodity A |  | Commodity B |  | Commodity C |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Price <br> (Rs) | Quantity(kg) | Price <br> (Rs) | Quantity(kg) | Price <br> (Rs) | Quantity(kg) |
| 1982 | 5 | 10 | 8 | 6 | 6 | 3 |
| 1984 | 4 | 12 | 7 | 7 | 3 | 4 |

Ans: $\mathrm{P}_{01}=83.59 ; \mathrm{Q}_{01}=120.6$
10. On the basis of the following information, calculate the Fisher's Ideal Index Number:

| Commodities | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price | Quantity | Price | Quantity |
| A | 2 | 40 | 6 | 50 |
| B | 4 | 50 | 8 | 40 |
| C | 6 | 20 | 9 | 30 |
| D | 8 | 10 | 6 | 20 |
| E | 10 | 10 | 5 | 20 |

11. Calculate Fisher's Ideal Index from the following data:

| Commodities | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Quantity | Price | Quantity | Price |
| A | 15 | 4 | 10 | 6 |
| B | 20 | 3 | 25 | 4 |
| C | 10 | 6 | 20 | 5 |
| D | 30 | 5 | 25 | 5 |

Ans: $\mathrm{P}_{\mathbf{0 1}}=\mathbf{1 4 9 . 1 5}$
12. Find Laspeyre's, Paasche's and Fisher's price and quantity index numbers from the following data:

| Commodities | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price (Rs) | Quantity (kg) | Price (Rs) | Quantity (kg) |
| A | 5 | 25 | 6 | 30 |
| B | 10 | 5 | 15 | 4 |
| C | 3 | 40 | 2 | 50 |
| D | 6 | 30 | 8 | 35 |

[C.A. Foundation, May 2007]
Ans: $\mathbf{P}_{\mathbf{0 1}}=\mathbf{1 1 1 7 . 7 4} ; \mathbf{1 1 2 . 7 3 ; 1 1 3 . 7 3 ;} \mathbf{Q}_{\mathbf{0 1}}=\mathbf{1 1 5 . 7 9} ; \mathbf{1 1 3 . 7 6} ; 114.77$
13. Given that $\sum p_{1} q_{1}=250, \sum p_{0} q_{0}=150$. Paasche's Index Number $=150$ and DorbishBowley's Index Number = 145, find out (i) Fisher's Ideal Index Number; and (ii) Marshall-Edgeworth's Index Number.
Ans: 144.9, 145.26
14. From the following data, construct a price index number of the group of four commodities by using Fisher's Ideal Formula:

| Commodities | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price per unit | Expenditure Rs | Price per unit | Expenditure Rs |
| A | 2 | 40 | 5 | 75 |
| B | 4 | 16 | 8 | 40 |
| C | 1 | 10 | 2 | 24 |
| D | 5 | 25 | 10 | 60 |

Ans: $\mathbf{P}_{\mathbf{0 1}}=219.1$
15. From the information given below, find the price index for the Year II with Year I as base by using Fisher's ideal index number formula:

| Commodities | Price (Rs)/unit |  | Total Value (Rs) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Year I | Year II | Year I | Year II |
| A | 35 | 36 | 700 | 756 |
| B | 31 | 40 | 465 | 480 |
| C | 30 | 32 | 240 | 320 |
| D | 20 | 22 | 40 | 44 |

[I.C.W.A (Inter) June 2001] Ans: $\mathbf{P}_{01}=111.34$
16. From the following data, construct Quantity Index Number by:
(i) Fisher's Method, and (ii) Marshall-Edgeworth's Method

| Commodities | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price (Rs) | Quantity (kgs) | Expenditure <br> (Rs) | Quantity (kgs) |
| A | 25 | 40 | 2,000 | 50 |
| B | 22 | 18 | 1,200 | 30 |
| C | 54 | 16 | 1,320 | 44 |
| D | 20 | 40 | 1,350 | 45 |
| E | 18 | 30 | 630 | 15 |

Ans: (i) 136.85 (ii) 134.94
17. From the data given below, calculate quantity index number for the year 2000 by using:
(i) Laspeyre's (ii) Paasche's and (iii) Fisher's formulae

| Commodities | Year 1999 |  | Year 2,000 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price | Value | Price | Value |
| A | 10 | 70 | 11 | 115.5 |
| B | 5 | 45 | 10 | 45 |
| C | 6 | 30 | 5 | 45 |

[C.S. Foundation Dec. 2000]


| 1997 | Price | 9 | 10 | 6 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1998 | Price | 12 | 12 | 9 | 15 |
| 1999 | Price | 15 | 14 | 12 | 20 |

Ans: Base $1996=100$; Price I No. for $1997=127.50$; Price I No. for $1999=207.50$ 24. 3The price relatives and weights of a set of commodities are given in the following table:

| Commodity | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| Price Relatives | 125 | 120 | 127 | 119 |
| Weights | $\mathrm{W}_{1}$ | $2 \mathrm{~W}_{1}$ | $\mathrm{~W}_{2}$ | $\mathrm{~W}_{2}+3$ |

If the sum of the weights is 40 and the index for the set is 122 , find the values of $W_{1}$ and $W_{2}$
Ans: $W_{1}=7$ and $W_{2}=8$
25. Given below are the prices and weights of given commodities for the year 1990, 1991 and 1992:

| Commodity | Weights | Price in Rupees |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 1990 | 1991 | 1992 |
| A | 20 | 12.00 | 18.00 | 24.00 |
| B | 15 | 3.00 | 6.00 | 15.00 |
| C | 10 | 12.50 | 18.75 | 25.00 |
| D | 40 | 10.00 | 30.00 | 50.00 |
| E | 15 | 4.50 | 9.00 | 13.50 |

Using either aggregative method or relative method, calculate the weighted price index numbers for 1991 and 1992, taking 1990 as base year.

Ans: Price indices based on Price Relatives are: for 1991 225, for 1992380

## Exercise B (Test of Index Number Formulae)

1. From the following data find the index numbers for the current year and the base year based on each other and show that the Geometric Mean makes it reversible but the Arithmetic Mean does not

| Commodity | Prices |  |
| :---: | :---: | :---: |
|  | Base Year | Current Year |
| A | 25 | 55 |
| B | 30 | 45 |

Ans: $P_{01}(A M)=185 P_{01}(G M)=181.66 ; P_{10}(A M)=56.06 ; P_{10}(G M)=55.05$ $\mathbf{P}_{\mathbf{0 1}} \times \mathbf{P}_{\mathbf{1 0}} \mathbf{( A M )} \neq \mathbf{1} ; \mathbf{P}_{\mathbf{0 1}} \times \mathbf{P 1 0} \mathbf{( G M )}=\mathbf{1}$
2. Compute Fisher's index number on the basis of the following data:

| Commodity | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \hline \text { Price (in’00 } \\ \text { Rs) } \end{gathered}$ | Expenditure(in'00 Rs) | Price (in'00 Rs) | $\begin{gathered} \hline \text { Expenditure(in'00 } \\ \text { Rs) } \end{gathered}$ |
| A | 3 | 25 | 10 | 60 |
| B | 1 | 10 | 2 | 24 |
| C | 4 | 16 | 8 | 40 |
| D | 2 | 40 | 5 | 75 |
| Also apply Factor Reversal Test to the above index number. |  |  |  |  |
| 3. Using the following data, show whether the time reversal test is satisfied by Fisher's price index |  |  |  |  |
| Commodity | - $\quad \mathrm{p}$ | $\mathrm{q}_{0}$ | $\mathrm{p}_{1}$ | $\mathrm{q}_{1}$ |
| A | - 12 | 30 | 14 | 20 |
| B |  | 20 | 15 | 16 |
| ( Ans: Yes |  |  |  |  |



|  | Goods A | Goods B | Goods C | Goods D | Goods E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1996 | 100 | 100 | 100 | 100 | 100 |
| 1997 | 90 | 125 | 134 | 118 | 133 |
| 1998 | 89 | 61 | 60 | 115 | 125 |
| 1999 | 112 | 200 | 80 | 93 | 140 |
| 2000 | 122 | 66 | 150 | 86 | 86 |

10. The price index of crude oil was 120 in 1997 with 1995 as base year and 130 in 1998 with 1997 as base. The price of crude further increased by $20 \%$ in 1999 over 1998 and decreased by $10 \%$ in 200 over 1999. It further decreased by $10 \%$ in 2001 over 2000. Obtain the chain base indices of crude prices of 2001 over 1995. Hints: Chain Indices - Chained to Base 1995

| Year | 1955 | 1997 | 1998 | 1999 | 2000 | 2001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chain Index. |  |  | $\frac{130}{100} \times 120$ | $\frac{100+20}{1000} \times 156$ | $\frac{100-10}{100} \times$ | $\frac{100-10}{100} \times 168.48$ |
| Chained to Base |  |  |  |  |  |  |
| $1995=100$ | 100 | 120 | $=156$ | $=187.20$ | $187.20=$ <br> 168.48 | $=151.63$ |

11. 1 Calculate the Chain Base index numbers from the data given below:

| Commodity | Price of Commodities (in Rs) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1991 | 1992 | 1993 | 1994 | 1995 |
| A | 2 | 3 | 4 | 2 | 7 |
| B | 3 | 6 | 9 | 4 | 3 |
| C | 4 | 12 | 20 | 8 | 16 |
| D | 5 | 7 | 18 | 11 | 22 |

Ans: 100, 197.50, 349.16, 170.70, 352.07
12. Calculate the chain base index numbers from the data given below:

| year | Price of Commodities (in Rs) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E |
| 1996 | 10 | 20 | 12 | 40 | 100 |
| 1997 | 12 | 22 | 14 | 45 | 110 |
| 1998 | 11 | 25 | 18 | 49 | 106 |
| 1999 | 14 | 28 | 10 | 43 | 102 |
| 2000 | 15 | 23 | 9 | 42 | 101 |

Ans: 100, 113.82, 122.74, 117.54, 111.88
Exercise D (Base Shifting)

1. The following are price index numbers (Base $1985=100$ )
$\begin{array}{lllllllllllll}\text { Year } & 1985 & 1986 & 1987 & 1988 & 1989 & 1990 & 1991 & 1992 & 1993 & 1994 & 1995\end{array}$ $\begin{array}{lllllllllllll}\text { Index No. } & 100 & 120 & 122 & 116 & 120 & 120 & 137 & 136 & 149 & 156 & 137\end{array}$ Shift the base to 1990 and recast the index numbers.
Ans: 83.33, 100, 101.67, 96.67, 100, 100, 114.17, 113.33, 124.17, 130.00, 114.17
2. The followings are the index number of wholesale prices of a certain commodity based on 1992:

| Year | 2012 | 2013 | 2014 | 2015 | 2016 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Index No. | 100 | 108 | 120 | 150 | 210 | Shift the base to 2014 and obtain new index numbers.

Ans: 93.33, 90, 100, 125, 175
3. In the following series of index numbers, shift the base from 2010 to 2013:

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Index No. | 100 | 105 | 110 | 125 | 135 | 180 | 195 | 205 |

Ans: 80, 84, 88, 100, 108, 144, 156, 164


```
(Base 2014 = 100)
Ans: Year 2010 2011 2012 2013 2014 2015
    I. No. }90.06 104.20 100.69 99.80 100 102.3
```

9. (a) A firm in a certain industry has an index of material prices based on movements in the prices of selected materials weighted by the quantities consumed in the base year. The price index series based on $2000=100$, for the year $2010-2015$ was as follows:

| 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 120.3 | 122.1 | 126.4 | 125.2 | 127.0 | 131.6 |

In 2015, the index was completely revised to take into account a change in the type of materials used. The new index, based on $2015=100$, showed the following values:

| 2015 | 2016 | 2017 |
| :---: | :---: | :---: |
| 100 | 106.3 | 109.4 |

(b) (i) Splice the new index to the old, i.e. splice 'forward;' (ii) Splice the old index to the new, i.e. splice 'backward'.
Ans: (i) $2016 \quad 2017$ (ii) $2010 \quad 2011 \quad 2012 \quad 2013 \quad 2014$

| 139.9 | 144 | 91.4 | 92.8 | 96.0 | 95.1 | 96.5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

10. What are the uses of 'base shifting' on an Index Number series? Prepare a spliced series of index numbers with 2013 as base from the following series:

| Years | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Index A | 100 | 120 | 135 |  |  |  |  |
| Index B |  |  | 100 | 115 | 125 | 145 |  |
| Index C |  |  |  |  |  | 100 | 110 |

$\begin{array}{llllllllll}\text { Ans: } & \text { Year } & 2008 & 2009 & 2010 & 2011 & 2012 & 2013 & 2014\end{array}$
Splicing Indices $A$
and $B$ to $C$ (Base $\begin{array}{llllllll}51.08 & 61.30 & 68.97 & 79.31 & 86.21 & 100 & 110\end{array}$ 2013)
11. Given the following data:

| Year | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Monthly Pay (Rs) | 10,500 | 11,000 | 11,500 | 12,500 | 13,500 | 14,000 | 14,500 |
| Price Index | 115 | 120 | 130 | 138 | 144 | 150 | 160 |

(i) Calculate the real monthly pay for each year.
(ii) In which year die the employees have the highest purchasing power?
(ii) What percentage of increase in the monthly pay for the year 2021 is required (if any) to compensate him with the purchasing power in the year of this highest real pay?
Ans: (i)
$\begin{array}{llllllllll}\text { Year } & 2015 & 2016 & 2017 & 2018 & 2019 & 2020 & 2021\end{array}$
Real Monthly pay 9130.43 9166.67 8846.15 9057.97 9375.009333 .339062 .50 (Rs)
(ii) Highest purchasing power corresponds to the year 2019, which is the year of highest real wages (Rs 9,375.00)
(iii) Required monthly increase in pay in $2021=\left(\frac{9375.00-9062.50}{9062.50}\right) \times \mathbf{1 0 0}=\mathbf{3 . 4 4 8 \%}$
12. Mean monthly wages ( X ) and cost of living index numbers $(\mathrm{Y})$ for the years 2010 to 2015 are given below:

| Year | $:$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rs | $\mathrm{X}:$ | 360 | 400 | 480 | 520 | 550 | 590 |
| Y: |  | 100 | 104 | 115 | 160 | 210 | 260 |

In which year the real income was (i) the highest (ii) the lowest?
Ans: (i) $\mathbf{2 0 1 2}$ (ii) 2015


(ii) Compute the amount of pay needed in 2016 to provide so that the buying powe equal to that enjoyed in 2013:

Ans: (i) Year 2013 Real Wages (Dollars)
i) $\frac{240}{120} \times 200=400$ Dollars

$$
\text { (ii) } \frac{240}{120} \times 200=400 \text { Dollars }
$$

201320142015 $166.67 \quad 462.50$

2016 140
19. The following data gives the average monthly income of a teacher and general index of price during 2010-17. Prepare the index number to show that change in the real income of the teacher and comment on price increase.

| Year | $:$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income | $:$ | 4,000 | 4,400 | 4,800 | 5,200 | 5,600 | 6,000 | 6,400 | 6,800 |
| Index | $:$ | 100 | 130 | 160 | 220 | 270 | 330 | 400 | 490 |

Ans: Real Income Indices (Base 2010 = 100):
100.00, $\quad 84.62,75.00,59.09,51.85,45.45,40.00,34.69$

Exercise E (Cost of Living Index)

1. Find the cost of living index for the following data:

| Group | Food | Clothing | Rent | Fuel and lighting | Miscellaneous |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Group Index | 180 | 150 | 100 | 110 | 80 |
| Weights | 140 | 42 | 49 | 56 | 63 |

Ans: 136
2. In the construction of a certain Cost of Living Index Number, the following group index numbers were found. Calculate the Cost of Living Index Number by using:
(i) The weighted arithmetic mean; and (ii) The weighted geometric mean

| Group | Food | Fuel and lighting | Clothing | House Rent | Miscellaneous |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Index <br> Number | 352 | 200 | 230 | 160 | 190 |
| Weights | 140 | 10 | 8 | 12 | 15 |

Ans: (i) 274.26 (ii) 261.1
3. A worker earned Rs 900 per month in 2010 . The cost of living index increased by $70 \%$ between 2010 and 2013. How much extra income should the worker have earned in 2013 so that he could buy the same quantities as in 2010 ?
Ans: Rs $\mathbf{1 2} \times\left[\left(\frac{\mathbf{1 7 0}}{\mathbf{1 0 0}} \times \mathbf{9 0 0}\right)-\mathbf{9 0 0}\right]=$ Rs 7,560
4. During a certain period the cost of living index number goes up from 110 to 200 and the salary of the worker is also increased from Rs 325 to Rs 550 . Does the worker really gain, and if so, by how much in real terms?
Ans: Loss of Rs 90.90
5. 13. Following information relating to workers in an industrial town is given.

Items of consumption Consumer Price Index in Proportion of expenditure 2020 on the items $(2010=100)$
(i) Food, drinks and tobacco

225 52\%
(ii) Clothing

175
(iii) Fuel and Lighting 155
(iv) Housing 250
(v) Miscellaneous 150

8\%
155 10\%
250 14\%

Average wage per month in 2010 was Rs 2,000. What should be the average wage per worker per month in 2020 in that town so that the standard of living of the workers does not fall below the 2010 level?
Ans: Rs 4,110.
6.

The adjoining table gives the cost of living index numbers for different groups with their


11. In calculating the cost of living index the following weights were used: Food $8 \frac{1}{2}$; Rent 2 ; Clothing $2 \frac{1}{2}$; Fuel and Light 1; Miscellaneous 11. Calculate the index number for a data when the percentage increase in prices of the various items over prices of July, $2018=$ 100 were $31,57,90,75$ and 88 respectively.

Ans: 152.2
12. In calculating a certain cost of living index number, the following weights were used. Food 15, Clothing 3, Rent 4, Fuel and Light 2, Miscellaneous 11. Calculate the index for a date when the average percentage increases in price of item in the various groups of the base period were $32,54,47,78$ and 58 respectively.

Suppose a business executive was earning Rs 2,050 in the base period. What should be his salary in the current period if his standard of living is to remain the same?

Ans: 141.76; Rs 2,906.08
13. The cost of living index uses the following weights:

Food 40, Rent 15, Clothing 10, Fuel 10, Miscellaneous 15. During the period 2010 - 15, the cost of living index raised from 100 to 205.83 . Over the same period the percentage rise in prices were:

Rent 60, Clothing 180, Fuel 75 and Miscellaneous 165. What is the percentage of change in the price of food?

Ans: 90
14. The relative importance of the following eight groups of family expenditure we found to be - Food 348, Rent 88, Clothing 97, Fuel and Light 65, House durable goods 71, Miscellaneous goods 35, Services 79, Drink and tobacco 217. The corresponding $\%$ increase in price for Oct. 2015 gave the following values - 25, 1, 22, 18, 14, 13, ?and 4. Calculate the percentage increase in group - services, if the percentage increase for whole group is 15.278 .

Ans: 11
15. From some given data, the retail price index based on five items, viz. Food, Rent and Rates, Fuel and Light, Clothing and Miscellaneous was calculated as 205. Percentage increases in prices over the base period are given below:

Rent and Rates 60, Clothing 210, Fuel and Light 120, Miscellaneous 130 Calculate the percentage increase in the Food Group, given that the weights of different items are as follows:
Food 60, Rent and Rates 16, Fuel and Light 8, Clothing 12, Miscellaneous 4, All items 100.

Ans: 92.3\% increase in food group
16. Calculate the cost of living index number form the following data:

| Group/Commodities | Weights | Group/Commodity Index Number |
| :--- | :---: | :---: |
|  | W |  |
| Food | 71 | 370 |
| Clothing | 3 | 423 |
| Fuel, etc. | 9 | 469 |
| House Rent | 7 | 110 |
| Miscellaneous | 10 | 279 |

[C.A. (Foundation), Nov. 2001]
Ans: 353.20
17. The subgroup indices of the consumer price index number of workers of an industrial town for the year 2013 (with base 2008) were:

| Food | Cloth | Fuel and Light | House Rent | Miscellaneous |
| :---: | :---: | :---: | :---: | :---: |
| 180 | 140 | 125 | 200 | 150 |

18. The weights of the various subgroups are $50,9,6,15$ and 20 respectively. It is proposed to fix industrial dearness allowance such that the employees are compensated fully for the rise in prices of food and house rent but only to the extent of $50 \%$ of increase in the prices of the rest of the sub-groups. What should be the dearness allowance expressed as percentage of wages?
Hint: Since the employees are compensated fully for the rise in price of food and house rent but only to the extent of $50 \%$ of increased in the prices of the rest of the subgroups, for clouting the C.O.L. Index (for giving compensation) were will take the index of cloth, fuel and light, and miscellaneous items as: $100+\frac{40}{2}=120,100+\frac{25}{2}=112.5$ and $100+\frac{50}{2}=125$, respectively. C.O.L. Index $=$ 162.5

Hence, the dearness allowance to be given to employees should be $62.55 \%$ of their wages in 2008.
19. The group indices and the corresponding weights for the working class cost of living index numbers in an industrial city for the years 2016 and 2020 are given below:

| Group | Weight |  | Group Index |
| :--- | :---: | :---: | :---: | :---: |
|  |  | 2016 | 2020 |
| Food | 71 | 370 | 380 |
| Clothing | 3 | 423 | 504 |
| Fuel, etc. | 9 | 469 | 336 |
| House Rent | 7 | 110 | 116 |
| Miscellaneous | 10 | 279 | 283 |

(a) Compute the cost of living indices for the two year 2016 and 2020.
(b) If a worker was getting Rs 3,000 per month in 2016 , do you think that he should be given some extra allowance so that he can maintain his 2016 standard of living? If so, what should be the minimum amount of this extra allowance?

Ans: (a) 353.20; 351.58 (b) No extra allowance should be given
20. Labour and capital are used in two different proportions to products A and B, but the price of each input is equal for both products. On the basis of the information given in the attached table, prepare, for the year 2020 separate price indices for labour and capital:

Weight for labour
Weight for capital

| Product A | Product B |
| :---: | :---: |
| 60 | 70 |
| 40 | 30 |
|  |  |
| 340 | 330 |

Ans: $P_{01}$ (Labour) = 300; $\mathbf{P}_{01}$ (Capital) $=400$
21. An enquiry into the budgets of the middle class families in a certain city in India gave the following information:

| Expenses on | Food | Fuel | Clothing | Rent | Misc. |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price in 2015 (Rs) | $35 \%$ | $10 \%$ | $20 \%$ | $15 \%$ | $20 \%$ |
| Price in 2016 (Rs) | 150 | 25 | 75 | 30 | 40 |

What is the cost of living index number of 2016 as compared with that of 2015 ?
Ans: 102.86
22. Using the formula $I x=\frac{\sum q_{0} p_{1}}{\sum q_{0} p_{0}} \times 100$, and find the consumer price index for 2020 with 2009 as base with the help of the following data. Interpret the Index Number so obtained.

| Item No. | Quantity consumed in | Price per unit in | Price per unit in |
| :---: | :---: | :---: | :---: |
|  | $2009\left(\mathrm{q}_{0}\right)$ | 2009 | 2009 |
|  |  | $\left(\mathrm{p}_{0}\right)$ | $\left(\mathrm{p}_{1}\right)$ |


23. Construct the consumer price index numbers for 2019 and 2020 from the indices given below:

| Year | Food | Rent | Clothing | Fuel | Misc. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2018 | 100 | 100 | 100 | 100 | 100 |
| 2019 | 102 | 100 | 103 | 100 | 97 |
| 2020 | 106 | 102 | 105 | 101 | 98 |

Assume the following weights for different groups:

| Food | Rent | Clothing | Fuel | Misc. |
| :---: | :---: | :---: | :---: | :---: |
| 60 | 16 | 12 | 8 | 4 |

Ans: For 2016: 101.44; For 2020: 104.52
24. Index of Industrial Production covers thre groups of industries. The index increased from 106.4 to 150.2 form one point of time to another. The index number of individual three groups of industries, over the same period, changed as follows: Mining and Quarrying form 102.0 to 144.1; Manufacturing from 106.5 to 146.6; Electricity from 110.4 to 189.9

Determine the weights for the individual groups of industries.
Ans: $\mathbf{( 9 . 9}, 81.2,8.9)=\mathbf{( 1 0 , 8 1}, 9)$
25. If the Consumer Price Index (for the same class of people and with same base year) is higher for Delhi than that for Mumbai, does it necessarily mean that Delhi is more expensive (for this class of people) than Mumbai. Give reasons in support of your answer.
26. Owing to change in prices, the consumer price index of the working class in a certain area roles in a month by one quarter of what it was before, to 225 . The index of food became 252 from 198, that of clothing from 185 to 2015, that of fuel and lighting from 175 to 195, and that of miscellaneous from 138 to 212 . The index of rent, however, remained unchanged at 15 . It was known that weight of clothing, rent, and fuel and lighting were the same. Find out the exact weight weights of all the groups.
Hints: Let $I_{1}$ and $I_{2}$ be the index number in the beginning of the month and at the end of the month respectively.

$$
\text { So } I_{2}=225 \text { and } I_{1}=\left(1+\frac{1}{4}\right) I_{1}=\frac{5}{4} I_{1} \Rightarrow I_{1}=\frac{4}{5} \times 225=180
$$

By forming and solving the two equations i.e. one at the beginning and one at the end, the following weights will be found:

Food: 54, Clothing: 10, Fuel and Lighting: 10, Rent: 10, Miscellaneous: 16
27. In a working class consumer price index number of a particular town the weights corresponding to different groups of items were as follows:

Food - 55, Fuel - 15, Clothing - 10, Rent - 8 and Miscellaneous - 12
In Oct. 2020, the DA was fixed by a mill of that town at 182 per cent for the workers which fully compensated for the rise in prices of food and rent but did not compensate for anything else. Another mill of the same town paid D.A. of 46.5 per cent which compensated for the rise in fuel and miscellaneous groups. It is known that rise in food is double than that of fuel and the rise in miscellaneous group is double than that of rent.

Find the rise in food, fuel, rent and miscellaneous groups.

Food: 317.14; Fuel: 158.57; Rent: 94.64; Miscellaneous: 189.28
28. The estimated per capita income for India in 1931-32 was Rs 65 . The estimated for 1972-73 was Rs 650. In 1972-73, every Indian was, therefore, 10times more prosperous than in 1931-32. Comment.

## Time Series Analysis <br> Exercise F

1. Discuss the relative merits and demerits of 'free-hand curve' method of studying trend. What point will you keep in mind in drawing such a trend curve?
With the help of graph paper, obtain the trend curve:

| Year | $:$ | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value $:$ | 64 | 82 | 97 | 71 | 78 | 112 | 115 |  |
| Year | $:$ | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 |  |
| Value $:$ | 131 | 88 | 100 | 146 | 150 | 120 |  |  |

2. Compute the values by the method of semi-averages from the data given below:
$\begin{array}{lllllllllll}\text { Year } & : & 2012 & 2013 & 2014 & 2015 & 2016 & 2017 & 2018 & 2019\end{array}$
No. of sheep (in lakhs) : $\quad 56 \quad 55 \quad 51 \quad 47 \quad 42 \quad 38 \quad 35$
Ans: Trend values (in lakhs) for the year 2012 to 2019 are: 59, 56, 50.5, 46.5, 41.5 37, 35, 32
3. The sales of a commodity in tones varied from January 2009 to December 2009 in the following manner:

| 280 | 300 | 280 | 280 | 270 | 240 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 230 | 230 | 220 | 200 | 210 | 200 |

Find a trend by the method of semi-average.
4. Fit a trend line form the following data by using semi-average method:

| Year $:$ | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Profit (in ‘000 lakhs) : | 100 | 120 | 140 | 150 | 130 | 200 |

Ans: By joining the points $(2014,120)$ and $(2017,160)$, we get the trend line.
5. Fit a straight line trend to the following data using the method of least squares and calculate the production for the year 2021:

| Year : | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Production ('000 tons) : | 83 | 92 | 74 | 90 | 166 |

Ans: $\mathrm{Y}=101$ +16.4X; $(\mathrm{X}-$ Origin = 1998); Estimated production for 2021 is 150.2
('000 tones)
6. Fit a straight line trend to the following data by Least Square Method:

| Year : | 2011 | 2013 | 2015 | 2017 | 2019 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Production : | 18 | 21 | 23 | 27 | 16 |

Specify the year of origin. Estimate production for the year 2018 and 2020.
Ans: $\mathrm{Y}=21+0.1 \mathrm{X}$ [Origin X : 2015]; $(Y)_{2018}=21.3 ;(Y)_{2020}=21.5$
7. Fit a straight line trend to the following data and estimate the value of output for the year 2017:

| Year : | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Production of steel: <br> (in million tons) | 60 | 72 | 75 | 65 | 80 | 85 | 95 |

Ans: $\mathrm{Y}_{\mathrm{c}}=76+4.86 \mathrm{X}$ (Origin 2010); $\mathrm{Y}=$ million tons: $(Y)_{2017}=110.02$ (million tons)
8. Below are given the figures of production (in thousand quintals) of a sugar factory:

| Year : | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Production (in '000 quintals): | 80 | 90 | 92 | 83 | 94 | 99 | 92 |

(i) Fit a straight line trend to these figures by the method of least squares.
(ii) Show the given data and the trend line on the graph paper;

## 

(iii) Estimate the production for 2022;
(iv) Find the slope of the straight line trend;
(v) Do the figures show a rising trend or a falling trend?
(vi) What does the different between the given figures and trend values indicate?

Ans: (i) Yc = 90 + 2X: Origin: 2016 ( $1^{\text {st }}$ July)
Trend values (' 000 quintals): 84, 86, 88, 90, 92, 94, 96 (iii) $(Y)_{2022}=102$ ('000 quintals); (iv) Slope $=2$ ('000 quintals) (v) Rising trend: since slope is positive
9. Fit a straight line trend to the time series data given below by 'least squares method' and predict the sales for the year 2020:

| Year (t) : | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales (in lakh Rs) (Y) : | 25 | 30 | 38 | 50 | 62 | 80 | 95 |

Ans: Straight line trend: $Y=54.29$ + 11.93X; $(X=t-1996)$ Estimated sales for 2020 are: $Y=[54.29+11.93(2020-1996)]=$ Rs 102.01 lakhs.
10. Fit a straight line trend to the following data by least squares method taking 2009 as the year of origin and estimate exports for the 2015:

| Year $:$ | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Export (in tonnes) : | 47 | 50 | 53 | 65 | 62 | 64 | 72 |

Ans: Straight line trend: $Y=59+4 x ;(X=t-1999)$ Estimated exports for 2015: $Y=59+4 \times 6=83$ tonnes.
11. Using the method of least squares, fit a straight line to the following data and find the trend values and short term fluctuations.

| Year : | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Values : | 232 | 226 | 220 | 180 | 190 | 168 | 162 | 152 | 144 |

Ans: Trend Values ( $\mathrm{Y}_{\mathrm{c}}$ ): 234, 222, 210, 198, 186, 174, 162, 150, 138
Short-term fluctuations: (Y-Yc): - 2, 4, 6, -18, 4, - 6, 0, 2, 6 (Assuming additive model)
12. You are given the exports of electronics goods from 2010 to 2019. Fit a linear trend to the exports data and estimated the expected exports for the year 2025;

| Year $\quad:$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Exports (crores Rs.) : | 11 | 16 | 13 | 18 | 22 | 20 |

Ans: $Y=11.529+1.063 X ;$ (Origin 1990): $\mathbf{Y}_{2025}=27.474$ (crores Rs.)
13. The following table shows the consumption of butter in a district in different years. Obtain the trend values by the method of least squares.

| Year $\quad:$ | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Consumption ('000 kgs.) : | 60 | 80 | 90 | 120 | 145 | 170 |

(b) Also obtain the monthly increase in consumption of butter.

Ans: (a) Yc = 110.83 + 11.07X; X = 2(t-2011.5)
Trend Values (in ' 000 kgs.): 55.48, 77.62, 99.76, 121.90, 144.04, 166.18
(b) Monthly increase in consumption of butter $=1.8450$ (' 000 kgs .) $=1845 \mathrm{kgs}$.
14. Fit a straight line trend equation by the method of least squares and estimate the value for 2019:

| Year : | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value : | 380 | 400 | 650 | 720 | 690 | 600 | 870 | 930 |

Ans: Yc = 655 + 35.83X; $\mathrm{X}=2$ ( t - 2013.5)
Trend Values: 404.19, 175.85, 547.51, 619.17, 690.83, 762.49, 834.15, 905.81; $Y_{2019}=1049.13$
15. The following data relate to the number of passenger cars (in millions) produced from 2013 to 2020:

| Year | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Fit a straight line trend by the method of least squares to the above time series data. Use your result to estimate the production in 2020 and compare it with actual production.

Ans: $Y=5.975+0.0512 X ; X=2(t-2016.5) ; Y_{2020}=6.3337$ millions.
16. In a study of its sales, a motor company obtained the following least square trend equations:
$Y=1,600+200 X$ (origin 2010, $X$ units $=1$ year; $Y=$ total number of units sold annually) The company has physical facilities to produce only 3,600 units a year and it believes that it is reasonable to assume that at least for the next decade the trend will continue as before.
(a) What is the average annual increase in the number of units sold?
(b) By what year will the company's expected sales have equaled its present physical capacity?
(c) Estimate the annual sales for 2015.

How much in excess of company's present physical capacity is this estimated value?
Ans: (a) 200 units (b) In 2020 (c) 4,600 units: Excess $=\mathbf{4 , 6 0 0}-\mathbf{3 , 6 0 0}=\mathbf{1 , 0 0 0}$ units
17. Convert the following annual trend equation for total sales of a company to a monthly trend equation:

$$
Y=162+15.8 X \text { (Origin: 2015; Scale: } 1 \text { units of } X=1 \text { year) }
$$

Forecast the sales for June, 2018 by the two equations. Compare your results.
Ans: $Y=13.5$ + 0.1097X; (Origin: 2015, $X$ unit = 1 month: $Y$ unit = monthly sales)
18. The trend of the annual sales of Bharat Aluminum Company is described by the following equation:
$\mathrm{Y}_{\mathrm{c}}=12+0.7 \mathrm{X}$ : (Origin: 2010; X unit $=1$ year and Y unit $=$ Annual production)
Step the equation down to a month to month basis and shift the origin to $1^{\text {st }}$ January 2010.

$$
\begin{aligned}
& \text { Ans: } Y_{c}=1+\frac{0.7}{144} X ;\left(\text { Origin: } 1^{\text {st }} \text { July 2010; } X \text { unit }=1 \text { month }\right) ; \\
& Y_{c}=0.9712+0.0048 X\left(\text { Origin: } 1^{\text {st }} \text { January, 2010 }\right)
\end{aligned}
$$

19. The trend equation for certain production is given by $: Y=3,600+288 t$ t; where
$\mathrm{Y}=$ Annual production in thousand tons: t: Time with origin, the year 2010 and unit = 1 year
Estimate the trend value of the production for September, 2014.
[I.C.W.A (Intermediate) June, 2000]
Hints: Monthly trend equation is given by:
20. $\mathrm{Y}=\frac{3,600}{12}+\frac{288}{144} \mathrm{t}=300+2 \mathrm{t}$ : Origin: $1^{\text {st }} \mathrm{July}, 2010$; t : Unit 1 month; Y : Monthly production For September 2014 i.e. $15^{\text {th }}$ September, 2014: $t=4 \times 12+2.5=50.5$ Estimated production for $2014=300+2 \times 50.5=401$ thousand tons.

## Exercise G

1. Using three-year moving averages, determine the trend and short-term fluctuations. Plot the original and trend values on the same graph paper:

| Year | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Production (in ‘000 tonnes) | 21 | 22 | 23 | 25 | 24 | 22 | 25 | 26 | 27 | 26 | Ans: Trend (3 yearly M.A.): $\begin{array}{lllllllll}22 & 23.3 & 24 & 23.7 & 23.7 & 24.3 & 26 & 26.3\end{array}$ Using additive model, short-term fluctuations are: $0,-\mathbf{0 . 3}, \mathbf{1 . 0}, \mathbf{0 . 3},-1.7,0.7,0,0.7$

2. Assuming an additive model, apply 3 year moving average to obtain the trend-free series for years 2 to 6 :

| Year | 1 | 2 | 34 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Exports (Rs lakhs) | 126 | 130 | 137 | 141 | 145 | 155 | 159 |
|  |  |  | $(44)$ |  |  |  |  |

```
lll
```

3. From the following data, calculate the trend values using four-yearly moving average

| Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Values | 506 | 620 | 1036 | 673 | 588 | 696 | 1116 | 738 | 663 | Ans: M.A. Values for 2011 to 2015 respectively are: 719, 738.75, 758.25, 776.375, 793.875

4. Assuming a four-year cycle, calculate the trend by the method of moving average from the following data relating to production of tea in a certain tea estate:

| Year | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Production (Kg.) | 464 | 515 | 518 | 467 | 502 | 540 | 557 | 571 | 586 | 612 |

[I.C.W.A (Intermediate) Dec' 1999] Ans: 4-yearly M.A.'s for 2013 to 2018 respectively are: 495.70, 503.60, 511.60, 529.50, 553.00, 572.50
5. From the given data, compute 'trend' and 'short-term fluctuations' by the Moving Average Method, assuming a four-yearly cycle and multiplicative model:

6. Eliminate trend by moving average method and comment:

| Year | $1^{\text {st }}$ Quarter | 2 $^{\text {nd }}$ Quarter | $3^{\text {rd }}$ Quarter | 4th $^{\text {th }}$ Quarter |
| :--- | :---: | :---: | :---: | :---: |
| 2015 | 40 | 35 | 38 | 40 |
| 2016 | 42 | 37 | 39 | 38 |
| 2017 | 41 | 35 | 38 | 42 |

Ans: M.A. Values (M.A.V.) and Trend Eliminated Values (T.A.V.) (Assuming multiplicative model) for $3^{\text {rd }}$ Quarter of 2015 to $2^{\text {nd }}$ Quarter of 2017, respective are: $\begin{array}{lllllllll}\text { M. A. V. } & 38.5 & 39.0 & 39.375 & 39.25 & 38.875 & 38.5 & 38.125 & 38.5\end{array}$ $\begin{array}{lllllllll}\text { T. E. V. } & 98.70 & 102.56 & 106.67 & 94.27 & 100.32 & 98.70 & 107.54 & 90.91\end{array}$
7. What is trend in a time series? The following table gives the annual sales (un Rs 1,000 ) of a commodity:

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales | 710 | 705 | 680 | 687 | 757 | 629 | 644 | 783 | 781 | 805 | 873 |

Determine the trend by calculating the 5 yearly moving averages.
8. [I.C.W.A. (Intermediate) June, 1995]

Ans: 5 yearly M.A. (Trend) for 2012 to 2018 are respectively: (Rs 1,000)

$$
\text { 707.80, } 691.60, \quad 679.40, \quad 700.00, \quad 718.80, \quad 728.40, \quad 777.00
$$

9. Find the trend for the following series using a three-year weighted moving average with weights 1, 2, 1 .

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
| Value | 2 | 4 | 5 | 7 | 8 | 10 | 13 |

Ans: 3.75, 5.25, 6.75, 8.25, 10.25
10. For the following series of observations, verify that the 2 -year centred moving average is equivalent to 3 -year weighted moving average with weights 1,2 , 1 respectively.

| Year | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value | 2 | 4 | 5 | 7 | 8 | 10 | 13 |

[I.C.W.A. (Intermediate) June, 2002]
Ans: M.A. Values for 2015 to 2019 are respectively: 3.75, 5.25, 6.75, 8.25, 10.25

11. For the following data, verify that the 5 -yearly moving weighted moving average trend values with weights $1,2,2,2,1$ respectively are equivalent to 4 -yearly centred moving average trend values.

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales (Rs lakhs) | 5 | 3 | 7 | 6 | 4 | 8 | 9 | 10 | 8 | 9 | 9 | Ans: M.A.'s for 2007 to 2013 are: 5.125, 5.625, 6.500, 7.250, 8.250, 8.875, 9.000

## Exercise 9.3

1. Compute the seasonal averages, and seasonal indices for the following time-series:

| Month | 2014 | 2015 | 2016 | Month | 2014 | 2015 | 2016 |
| :--- | :---: | :---: | :---: | :--- | :---: | :---: | :---: |
| Jan. | 15 | 23 | 25 | July | 20 | 22 | 30 |
| Feb. | 16 | 22 | 25 | Aug. | 28 | 28 | 34 |
| March | 18 | 28 | 35 | Sept. | 29 | 32 | 38 |
| April | 18 | 27 | 36 | Oct. | 33 | 37 | 47 |
| May | 23 | 31 | 36 | Nov. | 33 | 34 | 41 |
| June | 23 | 28 | 30 | Dec. | 38 | 44 | 53 |

[Hints: Use Method of Simple Average]
Ans: 70, 70, 90, 90, 100, 90, 80, 100, 110, 130, 120, 150.
2. Assuming no trend in the series, calculate seasonal indices for the following data:

| Year | Quarter |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | I | II | III | IV |
| 2014 | 78 | 66 | 84 | 80 |
| 2015 | 76 | 74 | 82 | 78 |
| 2016 | 72 | 68 | 80 | 70 |
| 2017 | 74 | 70 | 84 | 74 |
| 2018 | 76 | 74 | 86 | 82 |

[C.A. (Foundation) May, 1999]
Hints: Use the method of simple averages
Ans: Seasonal indices for the four quarters are: 98.43; 92.15; 108.90; 100.52
3. Find seasonal variations by the ratio-to-trend method from the data given below:

| Year | $1^{\text {st }}$ Quarter | $2^{\text {nd }}$ Quarter | $3^{\text {rd }}$ Quarter | $4^{\text {th }}$ Quarter |
| :--- | :---: | :---: | :---: | :---: |
| 2013 | 30 | 40 | 36 | 34 |
| 2014 | 34 | 52 | 50 | 44 |
| 2015 | 40 | 58 | 54 | 48 |
| 2016 | 54 | 76 | 68 | 62 |
| 2017 | 80 | 92 | 86 | 82 |

Ans: Straight line trend equation is given by: $Y=56+12 X$ Origin: 1995 ( $1^{\text {st }} \mathbf{J u l y}$ ): X units = 1 year : Y units : Average quarterly values. Seasonal Indices: 92.0, 117.4, 102.1, 88.5
4. Find the seasonal variations by the ratio to trend method from the data given below: Quarter

| Year | I | II | III | IV |
| :--- | :---: | :---: | :---: | :---: |
| 2011 | 60 | 80 | 72 | 68 |
| 2012 | 68 | 104 | 100 | 88 |
| 2013 | 80 | 116 | 108 | 96 |
| 2014 | 108 | 152 | 136 | 124 |
| 2015 | 160 | 172 | 172 | 164 |


| Quarter | $\mathrm{Q}_{1}$ | $\mathrm{Q}_{2}$ | $\mathrm{Q}_{3}$ | $\mathrm{Q}_{4}$ |
| :--- | :---: | :---: | :---: | :---: |
| S.I. (Adjusted) | 92.05 | 117.36 | 102.13 | 88.46 |

11. From the given ratios of observed values to trend values (\%), calculate seasonal indices. If sales for 2021 are expected to be Rs 2,000

| Years | Quarters |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | I | II | III | IV |
| 2017 | 80 | 95 | 80 | 110 |



## Fill in the blanks:

## Chapter - 1 (Collection of data)

1. There are $\qquad$ methods of collecting primary data.
2. There are $\qquad$ method of collecting secondary data.
3. Data are classified into $\qquad$ and $\qquad$ _.
4. $\qquad$ is a suitable method of collecting data when the informants are literates and spread over a vast area.
5. Data originally collected by an investigator are known as $\qquad$ -
6. Before a questionnaire is finalized, it should be $\qquad$ .
7. Process of assembling the primary data is called $\qquad$ of statistics.
8. Questionnaire method cannot be used when the informants are $\qquad$ .
9. Most of the statistical analysis rest on $\qquad$ data.
10. The data which is more reliable is known as $\qquad$ data.
11. The data which are collected from outside are known as ___ data.
12. The data which are collected from inside are known as $\qquad$ data.
13. The data which are collected from the source of origin are known as $\qquad$ data.
14. The data which are collected from published and unpublished sources are known as $\qquad$ data.
15. Person who conducts the statistical enquire is known as
16. The person who helps the investigator in collecting the information is known as $\qquad$ .
17. The persons from whom the statistical data are collected are known as $\qquad$ .
18. The investigation method in which the investigator himself personally goes to the sources of data is known as $\qquad$ method.
19. The $\qquad$ investigation method adopted when the original informants are either not found or found to be reluctant to provide information in writing.
20. The method of collecting data adopted by the New Paper or periodical agencies is known as
$\qquad$ —.
21. When a list of questions relating to the subject matter of the problem is sent to the informants then it is known as -------- method.
22. When the questionnaire contains only two types of answer i.e. Yes or No then it is known as
$\qquad$ -.
23. The questionnaire which contains three types of answers i.e. Yes or No or Don't know then it is known as $\qquad$ -.
24. The questionnaire which contains more than three alternatives answers is known as $\qquad$ -.
25. The strategy to test the questionnaire using a smaller sample compared to plan size is known as
$\qquad$ _.
26. That data which includes one piece of information is known as $\qquad$ -.
27. The set of data which includes information on two characteristics is known as $\qquad$
28. The set of data which comprises of three or more characteristics is known as $\qquad$
29. The data which are recorded in chronological order is known as $\qquad$ .
30. The data which not recorded in time sequence is known as $\qquad$ .

Answers:

| 1. | 5 | 2. | 3 |
| :--- | :--- | :--- | :--- |
| 3. | Primary, Secondary | 4. | Mailed Questionnaire |
| 5. | Primary Date | 6. | Pretested |
| 7. | Collection | 8. | Illiterates |
| 9. | Secondary data | 10. | Primary data |
| 11. | External data | 12. | Internal data |
| 13. | Primary data | 14. | Secondary data |
| 15. | Investigator | 16. | Enumerator |

(1)

| 17. | Respondent | 18. |
| :--- | :--- | :--- |
| 19. | Indirect oral investigation | 20. |
| 21. | Questionnaire | 22. |
| Dictonomous correspondent |  |  |
| 23. | Tricotomons | 24. |
| 25. | Pilot survey | 26. |
| 27. | Bivariate data | 28. |
| 29. | Time series data | Multi-variate data data |

## Chapter - 2 (Classification and Tabulation of data)

1. Classification is the $\qquad$ step in tabulation
2. In classification and tabulation $\qquad$ is the chief requisite and $\qquad$ is the chief teacher.
3. When data are observed $\qquad$ , the type of classification is known as chronological classification.
4. The number of observations corresponding to particular class is known as the $\qquad$ of that class.
5. $\qquad$ classification is done according to some characteristics that can be measured.
6. Classification, Seriation and Tabulation are $\qquad$ to each other.
7. Classification and tabulation facilitate further $\qquad$ _.
8. Statistical errors can be $\qquad$ through classification and tabulation.
9. A discrete variable can be presented in a $\qquad$ series.
10. When a series in which different value of a variable are shown in a discontinuous manner along with respective frequency is known as $\qquad$ series.
11. Arrangement of data into various groups or categories of homogeneous character is known as
$\qquad$ —.
12. When the data are classified on the basis of area or place then it is known as $\qquad$ _ classification.
13. A characteristic which varies in amount of magnitude under different time and place is known as
$\qquad$ _.
14. The variable which always takes an integral value viz. $10,15,20,12$, etc. is known as $\qquad$
15. The difference between the largest value and smallest value of a variable is known as $\qquad$ -.
16. The form of classification made by excluding the upper boundary of a class viz. ( $0-10$ ), 10 will be excluded is known as $\qquad$ _.
17. The form of classification made by including the upper boundary of a class viz. ( $0-10$ ) both 0 and 10 will be included is known as $\qquad$ —.
18. The number of items coming within or shown against a particular class is known as $\qquad$ .
19. The small lines put usually vertically as the representative of an item against a particular value e.g. IIII isknown as $\qquad$ _.
20. The frequency per unit of class interval or magnitude of a class is known as $\qquad$ .
21. Number of observations with similar or closely related values are put in separate bunches or groups and each group are in order of magnitude in a class is known as $\qquad$ _.
22. The arrangement of the different values of a variable in a successive manner is known as
$\qquad$ —.
23. An individual series arranged in order i.e. Ascending or Descending is known as $\qquad$
24. The row heading of a table is known as $\qquad$
Answers:

| 1. | First | 2. | Commonsense, Experience |
| :--- | :--- | :--- | :--- |
| 3. | Over a period of time | 4. | Frequency |
| 5. | Quantitative | 6. | Complementary |
| 7. | Analysis of data | 8. | Minimised |
| 9. | Continuous | 10. | Discrete |
| 11. | Classification | 12. | Geographical/Areal/spatial |
| 13. | Variable | 14. | Discrete variable |
| 15. | Range | 16. | Exclusive form |
| 17. | Inclusive form | 18. | Class frequency |

40. The mean of 10 observations is 20 and median is 15 . If 5 is added to each observation, the new mean is $\qquad$ and median is $\qquad$ _.
41. A distribution with two modes is called $\qquad$ and with more than two modes is called
42. Average suited for qualitative phenomenon is $\qquad$
43. If $25 \%$ of the observations lie above $80,40 \%$ of the observations are less then 50 and $70 \%$ are greater than 40, then $\qquad$ = 80; $\qquad$ = 50; $\qquad$ $=40$.
44. Relationship between Median, $\mathrm{Q}_{1}, \mathrm{Q}_{2}$ and $\mathrm{Q}_{3}$ is $\qquad$
45. $\mathrm{D}_{5}, \mathrm{P}_{80}$, Median., $\mathrm{D}_{7}$ and $\mathrm{P}_{50}$ are related by $\qquad$
46. Relationship between $\mathrm{D}_{4}, \mathrm{Q}_{2}, \mathrm{P}_{60}, \mathrm{P}_{75}$ and $\mathrm{Q}_{3}$ is $\qquad$
47. The empirical relationship between mean, median and mode for a moderately asymmetrical distribution is $\qquad$
48. If the maximum frequency is repeated then mode is located by the method of $\qquad$ -
49. The distance covered is fixed but speeds are varying and an average speed is to be calculated by using $\qquad$ mean
50. When depreciation is charged by diminishing balance method and an average rate of depreciation is to be calculated by using $\qquad$ mean.
51. If the A.M. of $X$ is 25 , then A.M. of $X+2$ will be $\qquad$ .
52. If the A.M. of $X$ is 18 , then A.M. of $X-2$ will be $\qquad$ .
53. If the A.M. of $X$ is 62 , then A.M. of $2 X+2$ will be $\qquad$ —.
54. If the A.M. of $X$ is 28 , then A.M. of $3 X-2$ will be $\qquad$ .
55. If the A.M. of $X$ is 26 , then A.M. of $\frac{X}{2}+5$ will be $\qquad$ .
56. The A.M. of 50 items is calculated to be 12 . If all the observations are increased by 3 , then the new A.M. will be $\qquad$ Answers:

| 1. | Mean | 2. | More |
| :---: | :---: | :---: | :---: |
| 3. | No | 4. | Extreme |
| 5. | Reciprocal | 6. | Combined |
| 7. | Divided | 8. | Zero |
| 9. | Equal | 10. | Zero |
| 11. | Positional | 12. | Frequency |
| 13. | 100 | 14. | 11 |
| 15. | Mode | 16. | Symmetrical |
| 17. | Smaller | 18. | Positional |
| 19. | Bi-modal | 20. | $5^{\text {th }}$ |
| 21. | 2 Mean | 22. | Open-end |
| 23. | Equal class interval | 24. | Median |
| 25. | Harmonic Mean | 26. | Geometric Mean |
| 27. | Symmetrical | 28. | Mode |
| 29. | Median Or Mode | 30. | Mean |
| 31. | Median or Mode | 32. | 1.7" |
| 33. | Mean | 34. | Median |
| 35. | Second | 36. | Very much affect |
| 37. | Open-end | 38. | Median, Mode |
| 39. | 9.47 | 40. | 25, 20 |
| 41. | Bi-modal, Multi-modal | 42. | Median |
| 43. | $\mathrm{Q}_{3}=80, \mathrm{P}_{40}=\mathrm{D}_{4}=50, \mathrm{P}_{30}=40$ | 44. | $\mathbf{Q}_{1} \leq \mathbf{Q}_{2}=\mathbf{M d} . \leq \mathbf{Q}_{3}$ |
| 45. | $\mathrm{D}_{5}=\mathrm{P}_{50}=\mathrm{Md} . \leq \mathrm{D}_{7} \leq \mathrm{P}_{80}$ | 46. | $\mathbf{D}_{4}<\mathbf{Q}_{2}<\mathbf{P}_{60}<\mathbf{P}_{75}=\mathbf{Q}_{3}$ |
| 47. | Mo. = 3Md. - 2M | 48. | Grouping |
| 49. | Harmonic | 50. | Geometric |



1. The sum of the squares of deviations of 10 observations taken from Mean 50 is 250 . The C.V. of the series will be $\qquad$ _.
2. The Mean and S.D. of a series are 30 and 60 respectively. By adding 2 to each variable, the new Mean = $\qquad$ and new S.D. = $\qquad$ .
3. If both the Mean and the Variance of a series are 25, the coefficient of S.D. $=$ $\qquad$ and C.V. =
$\qquad$ -.
4. If $25 \%$ of the items are less than 10 and $25 \%$ are more than 40 , the coefficient of Q.D. will be
$\qquad$ _.
5. The median and standard deviation are 20 and 4 respectively. If each item is increased by 2 , the median will be $\qquad$ and the S.D. will be $\qquad$ —.
6. Q. D. is an $\qquad$ measure of dispersion.
7. The $\qquad$ the Lorenz Curve is form the line of the equal distribution, the greater is the variability in the series.
8. Variance is equal to square of $\qquad$
9. Mean deviation is normally calculated by taking deviations from $\qquad$ .
10. Sum of the deviations is least when taken from $\qquad$ _.
11. Standard deviation is calculated by taking deviations from $\qquad$ -
12. If each item is added by 5 then the value of $S$. $D$. will be $\qquad$ .
13. If each item is multiplied by 3 , the value of $S$. $D$. will be $\qquad$ -.
14. If each item is multiplied by 2 , the value of Variance will be $\qquad$ _.
15. The measure of dispersion which is most affected by the values of extreme items is $\qquad$ .
16. The measure of dispersion which takes the middle $50 \%$ of the items is $\qquad$ _.
17. The half of Inter Quartile Range is known as $\qquad$ —.
18. The formula of coefficient of variation is $\qquad$ .

| 1. | 10 | 2. | 32 | 3. | $20,20 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4. | $60 \%$ | 5. | 22,4 | 6. | Absolute |
| 7. | Farther | 8. | Standard deviation | 9. | Median |
| 10. | Median | 11. | Arithmetic Mean | 12. | Unchanged |
| 13. | Multiplied by 3 | 14. | Multiplied by 4 | 15. | Range |
| 16. | Inter Quartile Range | 17. | Quartile Deviation | 18. | $\frac{\text { S.D. }}{\text { Mean }} \times 100$ |

UNIT - III

## Correlation Analysis:

| 31) When the two variables are plotted on a graph and it produces a straight line then the correlation is said to be _.$\qquad$ |  |  |  |
| :---: | :---: | :---: | :---: |
| 32) When the two variables are plotted on a graph and it produces a straight line moving upward from left to right then it is $\qquad$ correlation. |  |  |  |
| 33) When the two variables are plotted on a graph and it produces a straight line moving downward from left to right then it is $\qquad$ correlation. |  |  |  |
| 34) When the graph of two variables gives a curve line then it is known as $\qquad$ correlation. |  |  |  |
| Answer: |  |  |  |
| 1. | $\pm 1$ | 2. | Perfect positive |
| 3. | Perfect negative | 4. | No correlation |
| 5. | Unchanged | 6. | Unchanged |
| 7. | Unchanged | 8. | Unchanged |
| 9. | Independent | 10. | Independent |
| 11. | Origin, Scale | 12. | Spearman's |
| 13. | Equal | 14. | 6 |
| 15. | $\mathrm{r} \pm \mathrm{PE}_{\mathrm{r}}$ | 16. | $\mathrm{PE}_{\mathrm{r}}=0.6745 \times \frac{1-r^{2}}{\sqrt{n}}$ |
| 17. | $\mathrm{SEr}=\frac{1-r^{2}}{\sqrt{n}}$ | 18. | $r^{2}$ |
| 19. | Correlation | 20. | Conisation |
| 21. | Simple | 22. | Multiple |
| 23. | Positive | 24. | Negative |
| 25. | Perfect positive | 26. | Perfect negative |
| 27. | High degree positive | 28. | High degree negative |
| 29. | Low degree positive | 30. | Low degree negative |
| 31. | Linear or Perfect | 32. | Perfect Positive |
| 33. | Perfect Negative | 34. | Non-liner or imperfect |
| Regression Analysis: |  |  |  |
| 1) The Pearson's coefficient of correlation is the square root of the two ____. |  |  |  |
| 2) If $b_{x y}$ is positive then $b_{y x}$ will be |  |  |  |
| 3) If both $b_{x y}$ and $b_{y x}$ are positive then correlatio |  |  | coefficient will be |
| 4) The regression analysis measures |  |  | lations between X and Y |
| 5) The purpose of regression analysis is to study ___ _ between varia |  |  |  |
| 6) Lines of regression are ___ if $\mathrm{r}=0$ and they are ___ if $\mathrm{r}= \pm 1$. |  |  |  |
|  | The farther the two regression lines cut each other the $\qquad$ be the degree of correlation. |  |  |
|  | If the regression coefficient of X on Y and Y on X are -0.4 and -0.9 respectively then the value of correlation coefficient is $\qquad$ |  |  |
|  | If one regression coefficient is more than unity then the other must be $\qquad$ than |  |  |
| 10) The statistical tool with the |  |  | timate the value of <br> known as $\qquad$ |
| 11) The product of two regression coefficients must be _____ than 1 . |  |  |  |
| 12) The name of the variable whose value will be estimated is ___. |  |  |  |
| 13) The geometric mean of two regression coefficients is known as |  |  |  |
| 14) If $\mathrm{b}_{\mathrm{xy}}=0.5, \mathrm{r}=0.8$ and $\mathrm{V}_{\mathrm{y}}=16$, then $\sigma_{x}=\square$. |  |  |  |
| 15) If $b_{x y}=4$ then the value of $b_{y x}$ will be less than or equal to |  |  |  |
|  |  |  |  |
| (8) |  |  |  |


13. The percentage of money wage with respect to price index is known as $\qquad$ .
14. The base of BSE Sensex is $\qquad$ .
15. When the index number of constructed for a particular group of persons living in the society is known as $\qquad$ index.

Answers:

| 1. | Index Numbers | 2. | Base | 3. | Laspeyre's |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4. | Paasche's | 5. | Dorbish-Bowley | 6. | Fisher's |
| 7. | Marshall-Edgeworth | 8. | Un-weighted or simple | 9. | Quantity |
| 10. | Fixed Base | 11. | Chain Base | 12. | Splicing |
| 13. | Real wage | 14. | $1978-79$ | 15. | Cost of living |

Time Series Analysis:

1. The one that is very useful in forecasting in the short-term is $\qquad$ component.
2. The cyclical movements are due to $\qquad$ -.
3. An overall tendency of rise or fall in a time series is called $\qquad$ -.
4. Of the four components of a time series, the one that has primary uses for long term forecasting is $\qquad$ component.
5. The line obtained by the method of least square is known as the line of $\qquad$ ..
6. A time series consists of the data arranged in $\qquad$ order.
7. The equation of Gompertz Curve is of the form $\qquad$ _.
8. In trend equation $Y=A+b X$, $a$ is the $\qquad$ and $b$ is the $\qquad$ .
9. Comparable monthly data may be obtained by multiplying each of the values by
$\qquad$ and in a leap year by $\qquad$ .
10. A polynomial equation in the form of $Y=a+b X+c X 2+d X 3$ is called $a$ $\qquad$ .
11. The additive model of a time series is expressed as $\qquad$ .
12. When the difference between successive observations of a time series are constant or nearly so, the $\qquad$ may be an appropriate representation of trend equation.
13. Link relative method is one of the methods of determining $\qquad$ of a time series.
14. The tool which used to understand the underlying structure and function that produces the observation is known as $\qquad$ -.
15. The trend that gives the general tendency of the data for a long period is known as
$\qquad$ —.
16. Component of a time series which is defined as the repetitive and predictable movement around the trend line in one year or less is known as $\qquad$ .
17. The variation in a time series that usually lasts for two or more years is known as
$\qquad$ —.

Answers:

| 1. | Seasonal | 2. | Trade cycle | 3. | Secular tend |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4. | Trend | 5. | Best fit | 6. | Chronological |
| 7. | Y = abx | 8. | Y-intercept, Slope of trend | 9. | $30.4167,30.5$ |
| 10. | Second degree <br> parabola | 11. | Y = T = S + C + I | 12. | Straight line |
| 13. | Seasonal variation | 14. | Time Series Analysis | 15. | Secular trend |
| 16. | Seasonal variation | 17. | Cyclical variation |  |  |

UNIT - I

## Questions of 2 Marks and 3 Marks each

Two Marks Questions will be answered within 3 sentences each and Three Marks Questions will be answered within six sentences each

1. Explain the meaning of Descriptive Statistics.
2. Explain Statistics as Date and as Method.
3. What are the limitations of Statistics?
4. Explain the use of statistics in Commerce.
5. Explain the characteristics of Statistics.
6. What are the functions of Statistics?
7. What are the qualities of a Statistician?
8. Who is an investigator?
9. Who is an Enumerator?
10. What is primary data?
11. What is secondary data?
12. What do you mean by Pilot Survey?
13. What is time series data?
14. What do you mean by Cross Sectional Data?
15. What is Range?
16. What do you mean by class interval?
17. What is meant by class boundaries?
18. What is Tally Bar?
19. What do you mean by frequency density?
20. What do you mean by Relative Frequency?
21. What is open-end series?
22. Give the difference between Exclusive series and Inclusive series.
23. Give the differences between 'More than' and 'Less then' frequency distribution.
24. Distinguish between Univariate and Bi-variate series.
25. What do you mean by central value?
26. Give three features of a good average.
27. What is Arithmetic Mean?
28. What is simple Arithmetic Mean?
29. What do you mean by Weighted Arithmetic Mean?
30. What is combined Arithmetic Mean?
31. Write three properties of Arithmetic Mean.
32. Write two advantages of Arithmetic Mean.
33. Write two advantages of Arithmetic Mean.
34. What is Geometric Mean?
35. What is combined Geometric Mean?
36. Give three advantages of G.M.
37. Write two disadvantages of Geometric Mean.
38. For what purpose G.M. is used?
39. What is Harmonic Mean?
40. Give the uses of Harmonic Mean.
41. Give two advantages of Harmonic Mean.
42. Give three limitations of H.M.
43. What are the relationship between A.M., G.M. and H.M.?
44. What is median?
45. Write three advantages of Median.
46. Write three limitations of Median.
47. What do you mean by Quartiles?
48. What is Decile?
49. What is percentile?
50. What is Mode?
51. Give three advantages of Mode.
52. Give three disadvantages of Mode.
53. What is the relationship between Mean, Median and Mode?
54. What do you mean by Emperical Mode?

## PRACTICAL PROBLEMS:

1. Calculate average bonus paid per member from the following data:

| Bonus (in Rs) | 50 | 60 | 70 | 80 | 90 | 100 | 110 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons | 1 | 3 | 5 | 7 | 6 | 2 | 1 |

Ans: Rs 79.60
2. Peter travelled by car for 4 days. He drove 10 hours each day. He drove: first day at the rate of 45 km per hour; second day at the rate of 40 km per hour; third day at the rate of 38 km per hour and fourth day at the rate of 37 km per hour. What was his average speed?

Ans: 40 km p. h .
3. Typist A can type a letter in 5 minutes, typist B in 10 minutes and typist C in 15 minutes. What is the average number of letters types per hour per typist?

Ans: Required average $=(12+6+4) / 3=7.33$
4. A taxi ride in a city costs one rupee for first kilometre and sixty paise for each additional kilometre. The cost of each kilometer is incurred at the beginning of the kilometre, so that the rider pays for a whole kilometer. What is the average cost for $2 \frac{3}{4}$ kilometre?

Ans: Average cost for $2 \frac{3}{4}$ kilometre $\left.=\mathbf{( 1 0 0}+\mathbf{6 0}+60\right) \times \frac{4}{11}$ paise $=\mathbf{8 0}$ paise
5. The mean weight of student in a group of 6 students is 119 lbs. The individual weights of five of them are $115,109,129,117$ and 114 lbs . What is the weight of the sixth student?

Ans: 130 lbs.
6. Average marks in Statistics of 10 students of a class was 68. A new student took admission with 72 marks, whereas two existing students left the college. If the marks of these students were 40 and 39 , find the average marks of the remaining students.

Ans: $\mathbf{7 5 . 7 8}$ (approx.)
7. Shri Narendra Kumar has invested his capital in three securities, namely RELIANCE Ltd.. TISCO and SATYAM: Rs 40,000 , Rs 50,000 and Rs 80,000 respectively. If he collects dividends of Rs 10,000 from each company, compute his average return from three securities.

Ans: 17.5\%
8. Twelve persons gambled on a certain night. Seven of them lost at an average rate of Rs 10.50 while five gained at an average of Rs 13.00 . Is the information given above correct? If not, why?

Ans: Information is incorrect.
9. Goals scored by a hockey team in successive matches are $5,7,4,2,4,0,5$ and 3 . What is the number of goals, the team must score in $10^{\text {th }}$ match in order that the average comes to 4 goals per match.

Ans: 5
10. The sum of deviations of a certain number of observations measured from 4 is 72 and the sum of the deviations of the same value form 7 is -3 . Find the number of observations and their mean.
11. The daily average sales of a store were Rs 2,750 for the month of Feb. 1996. During the month, the highest and the lowest sales were Rs 8,950 and Rs 580 respectively. Find the average daily sales if the highest and the lowest sales are not taken into account.

Ans: Rs 2,600.74
12. Two variables $X$ and $Y$ are related by: $Y=(X-5) / 10$ and each of them has 5 observations. If the man of X is 45 , find mean of Y .

Ans: 4
13. The average salary of 49 out of 50 employees in a firm is Rs 100 . The salary of $50^{\text {th }}$ employee is Rs 97.50 more than the average salary of all the 50 workers. Find the mean salary of all the employees of the firm.
14. The mean of 99 items is 55 . The value of $100^{\text {th }}$ item is 99 more than the mean of 100 items. What is the value of $100^{\text {th }}$ item?
15. The mean of 200 items was 50 . Later on it was discovered that two items were wrongly read as 92 and 8 instead of 192 and 88 . Find out the correct mean.
16. The average daily income for a group of 50 per sons working in a factory was calculated to be 169. It was later discovered that one figure was mis-read as 134 instead of the correct value 143. Calculate the correct average income.
17. The average marks of 80 students were found to be 40 . Later, it was discovered that a score of 4 was misread as 84 . Find the correct mean of 80 students.
18. The mean age of a combined group of men and women is 30 years. If the mean age of the group of men is 32 and that of the group of women is 27 , find out the percentage of men and women in the group.
19. The mean annual salary of all employees is a company is Rs 25,000 . The mean salary of male and female employees is Rs 27,000 and Rs 17,000 respectively. Find the percentage of males and females employed by the company.
20. If the means of two groups of $m$ and $n$ observations are 40 and 50 respectively, and the combined mean of two groups is 42 , find the ratio $m: n$.
21. The mean marks obtained by 300 students in the subject of Statistics are 45 . The mean of the top 100 of them was found to be 70 and the mean of the last 100 was known to be20. What is the mean of the remaining 100 students?
22. The mean hourly wage of 100 labourers working a factory, running two shifts of 60 and 40 workers respectively, is Rs 38 . The mean hourly wage of 60 labourers working in a morning shift is Rs 40 . Find the mean hourly wage of 40 labourers working in the evening shift.
23. There are three section of B. Com. $1^{\text {st }}$ year in a certain college. The number of students in each section and the average marks obtained by them in the Statistics paper in the annual examination are as follows:

| Section | Average marks in Statistics | No. of Students |
| :---: | :---: | :---: |
| A | 75 | 50 |
| B | 60 | 60 |
| C | 55 | 50 |

Find the average marks obtained by the students of all the sections taken together.
24. B. Com. (Pass) III year has three Sections A, B and C with 50,40 and 60 students respectively. The mean marks for the three sections were determined as 85,60 and 65 respectively. However, marks of a student of section A were wrongly recorded as 50 instead of zero. Determine the mean marks of all the three sections put together.
25. The mean monthly salary paid to 77 employees in a company was Rs 78 . The mean salary of 32 of them was Rs 75 and that of other 25 was 82 . What was the mean salary of the remaining?
26. Define the weighted arithmetic mean of a set of numbers. Show that it is unaffected if all the weights are multiplied by some common factor.
27. A contractor employs three types of workers male, female and children. To a male worker he pays Rs 16 per hour, to a female worker Rs 13 per hour and to a child worker Rs 10 per hour. What is the average wage per hour paid by the contractor if the number of male, female and children is 20,15 and 5 respectively?
28. Find the harmonic mean of the numbers: $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1$
[I.C.W.A. (Foundation) Jun 2004, Dec; 2002]
Ans: $\frac{1}{3}$
29. If each of 3,48 and 96 occurs once and 6 occur thrice, verify that geometric mean is greater than harmonic mean.

Ans: G.M. = 12; H.M. = 6.94; G.M. > H.M.
30. Show the weighted harmonic mean of the first $n$ natural numbers, where the weights are equal to the corresponding numbers, is given by $(n+1) / 2$.
31. An aeroplane flies around a square the sides of which measuring 100 km . each. The aeroplane covers at a speed of 100 km . per hour first side, at 200 km . per hour the second side, at 300 km . per hour the third side and at 400 km . per hour the fourth side. Use the correct mean to find the average speed around the square.

Ans: 192 km. p.h.
32. Four factories emit a kilogram of pollutant each in $4,5,8$ and 12 days respectively. What is the average rate of pollutant discharge? Use you answer to calculate the total pollutant discharged by the four factories in one week.
Hint: Find H.M. of 4, 5, 8, 12
Ans: 1kg. pollutant in 480/79 days per factory. Total pollutant discharged by four factories per week $=\frac{79}{480} \times 4 \times 7=4.608 \mathrm{~kg}$.
33. A railway train runs for 30 minutes at a speed of 40 miles an hour and the, because of repairs of the tract runs for 10 minutes at a speed of 8 miles an hour, after which it resumes its previous speed and runs for 20 minutes except for a period of 2 minutes when it had to run over a bridge with a speed of 30 miles per hours. What is the average speed?
Hints: Average speed $=$ Total Distance covered $\div$ Total time taken
$=\left[\left(\frac{40}{60} \times 30+\frac{8}{60} \times 10+\frac{40}{60} \times 18+\frac{30}{60} \times 2\right) \div(30+10+20]\right.$ m.p.h
34. A cyclist covers his first three kilometres at an average speed of 8 km . per hour, another 2 kms. at 9 km . per hour and the last 2 kms . at 4 km . per hour. Find the average speed for the entire journey.

Ans: 6.38 kms. Per hour
35. If $X$ travels 8 kms . at 4 km . per hour; 6 kms . at 3 km . per hour and 4 kms . at 2 km . per hour, what would be the average rate per hour at which he travelled?

Ans: Weighted H.M. $=3$ km. p.h.
36. A man travelled by car for 3 days. He covered 480 km . each day. On the first day he drove for 10 hours at 48 km . an hour; on the second day he drove for 12 hours at 40 km . an hour and on the last day he drove for 15 hours at 32 km . an hour. What was his average speed?

Ans: $\mathbf{3 8 . 9 1 9} \mathbf{~ k m} . \mathbf{p . h}$.
37. Kishore travels 900 kms . by train at an average speed of 60 km . per hour; 3,000 km. by steamship at an average of 25 km . per hour; 400 kms . by aeroplane at 350 km . per hour; and finally 15 kms . by bus at 25 km . per hour. Calculate his average speed for the entire journey.

Ans: $\mathbf{3 1 . 5 5 6} \mathbf{~ k m}$. p. h.
38. 32. A man travels from Agra to Dehradun covering 204 miles at a mileage rate of 10 miles per gallon of petrol and via Ghaziabad with an additional journey of 40 miles at the rate of 15 miles per gallon. Find the average mileage per gallon.

Ans: $\mathbf{1 0 . 5 8}$ miles per gallon.
39. The consumption of petrol by a motor was a gallon for 20 miles while going up from plains to hill station and a gallon for 24 miles while coming down. What particular average would you consider appropriate for finding the average consumption in miles per gallon for up and down journey, and why?

Ans: Harmonic Mean $\mathbf{=} \mathbf{2 1 . 8 2}$ miles per gallon.
40. A man having to drive 90 kilometres wishes to achieve an average speed of 30 kilometres per hour. For the first half of the journey he averages only $20 \mathrm{~km} . \mathrm{p}$. h. What must be his average for the second half of the journey if his overall average is to be $30 \mathrm{~km} . \mathrm{p} . \mathrm{h}$.

Ans: $\mathbf{6 0} \mathbf{~ k m}$. p.h.
41. An aeroplane travels distances of $d_{1}, d_{2}$ and $d_{3} k m s$. at speed $V_{1}, V_{2}$ and $V_{3} k m$. per hour respectively. Show that the average speed (V) is given by:

$$
\frac{d_{1}+d_{2}+d_{3}}{V}=\frac{d_{1}}{V_{1}}+\frac{d_{2}}{V_{2}}+\frac{d_{3}}{V_{3}}
$$

42. A person purchases one kilogram of cabbage from each of the four places at the rate of 20 kg ., $16 \mathrm{~kg} ., 12 \mathrm{~kg}$. and 10 kg . per 100 rupee respectively. On the average how many kg.of cabbage has he purchased per 100 rupee?
43. If you spend Rs 100 per week on apples and the price of apples for three weeks is Rs 25 , Rs 20 and Rs 10 per kilogram, what is the average price of apples for you?

Ans: Rs 15.79
44. In a certain office a letter is types by A in 4 minutes. The same letter is types by B, C and D in $5,6,10$ minutes respectively. What is the average time taken in completing one letter? How many letter do you expect to be typed in one day comprising of 8 working hours.
Ans: H.M. = 5.58 minutes per letter; Letters type in $\mathbf{8}$ hours ( $\mathbf{4 8 0}$ minutes) $=\frac{\mathbf{4 8 0}}{\mathbf{5 . 5 8}} \mathbf{= 8 6}$
45. A scooterist purchased petrol at the rate of Rs 24 , Rs 29.50 and Rs 36.85 per litre during three successive years. Calculate the average price of petrol:
(i) If he purchased 150, 180 and 195 litres in the respective years; and
(ii) If spent Rs 3,850 , Rs 4,675 and Rs 5,825 in three years

Give support to your answer.
Hints: Average price of petrol/litre $=\frac{\text { Total money spent on petrol }}{\text { Total petrol consumed in litres }}$
(i) Weighted A.M. of prices the weights being the quantities of petrol purchased.
(ii) Weighted H.M. of prices, the weights being money spent on petrol.

Ans: (i) Rs 30.65/ litre (ii) Rs 30/litre (approx.)
46. Define Arithmetic Mean, Harmonic Mean and Geometric Mean for a set of $n$ observations and state the relationship between them.
Ans: $A \geq G \geq \mathbf{H}$; the sign of equality holds if and only if all the observations are equal.
Show the relationship between arithmetic mean and harmonic mean for the variable X , which can take the values a and b such that $\mathrm{a}, \mathrm{b}$ are non-negative integers.

$$
\text { Ans: } \mathrm{A} \times \mathrm{H}=\left(\frac{a+b}{2}\right)\left(\frac{2 a b}{a+b}\right)=\mathrm{ab}=G^{2}
$$

47. If for two number, the arithmetic mean is 25 and the harmonic mean is 9 , what is the geometric mean of the series.
48. If A.M. of two numbers is 17 and G.M. is 15 , find the H.M. of these numbers.

Ans: G.M. = 15
49. Comment on the following: "The G.M. and A.M. of a distribution are 27 and 30. Then H.M. is 26."
50. State giving reasons which average will be more appropriate in the following cases:
(i)The distribution has open-end classes.

Ans: (i) Md. or Mo.
(ii) The distribution has wide range of variations.

Ans: G.M.
51. Find the G.M. of $1,2,3, \frac{1}{2}, \frac{1}{3}$. What will be the geometric mean is ' 0 ' is added to this set of values?
[I.C.W.A (Foundation) June 2003]
Ans: 1 and 0
52. Find the geometric mean of: $1,7,29,18,65,91$ and 103.

Ans: 20.62
53. Calculate the geometric mean of the data: 1, 7, 29, 92, 115 and 375

Ans: $\mathbf{3 0 . 5 0}$
54. If population of a city doubled itself in twenty years, is it correct to say that the rate of growth has been 5\% per annum?

Ans: No, r = 3.5\%
55. The population of a city was $1,00,000$ in 2005 and $1,44,000$ a decade later. Estimate the population at the middle of the decade.

Ans: 1,20,000
56. The population of India in 2011 and 2021 were 361 and 439 million respectively.
a. What was the average percentage increase per year during the period?
b. If the average rate of increase from 2011 to 2021 remains the same, what would be the population in 2031?

## Ans: (i) 2\% (ii) $\mathbf{5 3 3 . 8 5}$ million

57. The population of a country increased by 20 per cent in the first decade and by 30 per cent in the second decade and by 45 per cent in the third decade. Determine the average decennial growth rate of population.

Ans: 31.3\%
58. A machine depreciates by $40 \%$ in the first year, by $25 \%$ in the second year and by $10 \%$ per annum for the next three years, each percentage being calculated on the diminishing value. What is the average percentage of depreciation for the entire period.

Ans: 20\%
59. An income tax assessee depreciated the machinery of his factory by 20 per cent in each of the first two years and 40 per cent in the third year. How much average depreciation relief should he claim from the taxation department?

Ans: 27.32\%
60. (b) A businessman depreciated the machinery of his factory by $20 \%$ in the first two years and $40 \%$ in the third year. What is the average depreciation for the three years?

Ans: G.M. = 27.32\%
61. An economy grows at the rate of $2 \%$ in the first year, $2.5 \%$ in the second year, $3 \%$ in the third year, $4 \%$ in the fourth $\qquad$ and $10 \%$ in the tenth year. What is the average rate of growth on the economy?

Ans: 5.6\% p.a.
62. The annual rates of growth achieved by a nation for 5 years are $5 \%, 7.5 \%, 2.5 \%, 5 \%$ and $10 \%$ respectively. What is the compound rate of growth for the 5 year period?

Ans: 5.9\%
63. The number of divorces per 1,000 marriage in a big city in India increased from 96 in 2010 to 120 in 2020. Find the annual rate of increase of the divorce rate for the period 2010 to 2020.

Ans: $\mathbf{r}=\mathbf{2 . 2 6 \%}$
64. If arithmetic mean and geometric mean of two values are 10 and 8 respectively, find the values.

Ans: 16, 4
65. 18. A man gets three successive annual raises in salary of $20 \%, 30 \%$ and $25 \%$ respectively, each percentage being reckoned in his salary at the end of the previous year. How much better or worse would he have been if he had been given three annual raises of $25 \%$ each, reckoned in the same way.
Ans: The man would be better in the second case by $0.31 \%$ of his starting salary in the $1^{\text {st }}$ year.
66. The geometric mean of 4 items is 100 and of another 8 items is 3.162 . Find the geometric mean of the 12 items.

Ans: 10
67. Geometric mean of $n$ observations is found to be G. How will you find the correct value of the Geometric Mean if some of the values used in its calculation are found to be wrong and should be replaced by correct values?
68. Geometric mean of 2 numbers is 15 . If by mistake one figure is taken as 5 , instead of 3 , find the correct geometric mean.

Ans: 11.62
69. The geometric mean of four values was calculated as 16. It was later discovered that one of the values was recorded as 32 when, in fact, it was 162. Calculate the correct geometric mean.

Ans: 24
70. Define simple and weighted geometric mean of a given distribution.
71. The weighted geometric mean of three numbers 229,275 and 125 is 203 . The weights for $1^{\text {st }}$ and $2^{\text {nd }}$ numbers are 2 and 4 respectively. Find the weight of the third.

Ans: 3
72. The weighted geometric mean of the four numbers $9,25,17$ and 30 is 15.3 . If the weights of the first three numbers are 5,3 and 4 respectively, find the weights of the fourth number.

Ans: 2 (approx.)
73. Define Harmonic Mean and discuss its merits and demerits. Under what situations would you recommend its use.
74. In a moderately skewed distribution:

Arithmetic mean $=24.6$ and the mode $=26.1$. Find the value of the median and explain the reason for the method employed

Ans: Median = $\mathbf{2 5 . 1}$
75. In a moderately asymmetrical distribution the value of median is 42.8 and the value of mode is 40 . Find the mean.

Ans: Mean = 44.2
76. In a moderately asymmetrical distribution the value of mean is 75 and the value of mode is 60 . Find the median.

Ans: Median = 70
77. In a moderately skewed distribution Arithmetic mean $=24.6$ and the mode $=26.1$. Find the value of the median and explain the reason for the method employed.

Ans: Median = 25.1
78. In a moderately asymmetrical distribution the value of median is 42.8 and the value of mode is 40 . Find the mean.

Ans: Mean = 44.2
79. In a moderately asymmetrical distribution the value of mean is 75 and the value of mode is 60 . Find the median.

Ans: Median = $\mathbf{7 0}$
80. Find the G.M. of $1,2,3, \frac{1}{2}, \frac{1}{3}$. What will be the geometric mean is ' 0 ' is added to this set of values?
[I.C.W.A (Foundation) June 2003] Ans: 1 and 0
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89. (a) An economy grows at the rate of $2 \%$ in the first year, $2.5 \%$ in the second year, $3 \%$ in the third year, $4 \%$ in the fourth $\qquad$ and $10 \%$ in the tenth year. What is the average rate of growth on the economy?
Ans: 5.6\% p.a.
(b) The annual rates of growth achieved by a nation for 5 years are $5 \%, 7.5 \%, 2.5 \%, 5 \%$ and $10 \%$ respectively. What is the compound rate of growth for the 5 year period?

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90. The number of divorces per 1,000 marriage in a big city in India increased from 96 in 2010 to 120 in 2020. Find the annual rate of increase of the divorce rate for the period 2010 to 2020.

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Ans: 10
94. (a) Geometric mean of $n$ observations is found to be G. How will you find the correct value of the Geometric Mean if some of the values used in its calculation are found to be wrong and should be replaced by correct values?
(b) Geometric mean of 2 numbers is 15 . If by mistake one figure is taken as 5 , instead of 3 , find the correct geometric mean.
Ans: 11.62
(c) The geometric mean of four values was calculated as 16. It was later discovered that one of the values was recorded as 32 when, in fact, it was 162 . Calculate the correct geometric mean.

Ans: 24
95. Define simple and weighted geometric mean of a given distribution.

The weighted geometric mean of three numbers 229,275 and 125 is 203. The weights for $1^{\text {st }}$ and $2^{\text {nd }}$ numbers are 2 and 4 respectively. Find the weight of the third.

Ans: 3
96. The weighted geometric mean of the four numbers $9,25,17$ and 30 is 15.3 . If the weights of the first three numbers are 5,3 and 4 respectively, find the weights of the fourth number.

Ans: 2 (approx.)
97. Find the harmonic mean of the numbers: $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1$
[I.C.W.A. (Foundation) Jun 2004, Dec; 2002]
98. If each of 3,48 and 96 occurs once and 6 occur thrice, verify that geometric mean is greater than harmonic mean.

Ans: G.M. = 12; H.M. = 6.94; G.M. > H.M.
99. Show the weighted harmonic mean of the first $n$ natural numbers, where the weights are equal to the corresponding numbers, is given by $(n+1) / 2$.
100. An aeroplane flies around a square the sides of which measuring 100 km . each. The aeroplane covers at a speed of 100 km . per hour first side, at 200 km . per hour the second side, at 300 km . per hour the third side and at 400 km . per hour the fourth side. Use the correct mean to find the average speed around the square.

Ans: 192 km. p.h.
101. Four factories emit a kilogram of pollutant each in $4,5,8$ and 12 days respectively. What is the average rate of pollutant discharge? Use you answer to calculate the total pollutant discharged by the four factories in one week.
Hint: Find H.M. of 4, 5, 8, 12
Ans: 1kg. pollutant in 480/79 days per factory. Total pollutant discharged by four factories per week $=\frac{79}{480} \times 4 \times 7=4.608 \mathrm{~kg}$.
102. A railway train runs for 30 minutes at a speed of 40 miles an hour and the, because of repairs of the tract runs for 10 minutes at a speed of 8 miles an hour, after which it resumes its previous speed and runs for 20 minutes except for a period of 2 minutes when it had to run over a bridge with a speed of 30 miles per hours. What is the average speed?
Hints: Average speed $=$ Total Distance covered $\div$ Total time taken
$=\left[\left(\frac{40}{60} \times 30+\frac{8}{60} \times 10+\frac{40}{60} \times 18+\frac{30}{60} \times 2\right) \div(30+10+20]\right.$ m.p. $h$
103. A cyclist covers his first three kilometres at an average speed of 8 km . per hour, another 2 kms. at 9 km . per hour and the last 2 kms . at 4 km . per hour. Find the average speed for the entire journey.

Ans: 6.38 kms. Per hour
104. If $X$ travels 8 kms . at 4 km . per hour; 6 kms . at 3 km . per hour and 4 kms . at 2 km . per hour, what would be the average rate per hour at which he travelled?
Ans: Weighted H.M. = $\mathbf{3} \mathbf{~ k m}$. p.h.
105. A man travelled by car for 3 days. He covered 480 km . each day. On the first day he drove for 10 hours at 48 km . an hour; on the second day he drove for 12 hours at 40 km . an hour and on the last day he drove for 15 hours at 32 km . an hour. What was his average speed?

Ans: $\mathbf{3 8 . 9 1 9} \mathbf{~ k m} . \mathbf{p . h}$.
106. Kishore travels 900 kms . by train at an average speed of 60 km . per hour; 3,000 km. by steamship at an average of 25 km . per hour; 400 kms . by aeroplane at 350 km . per hour; and finally 15 kms . by bus at 25 km . per hour. Calculate his average speed for the entire journey.

Ans: $\mathbf{3 1 . 5 5 6} \mathbf{~ k m}$. p. h.
107. A man travels from Agra to Dehradun covering 204 miles at a mileage rate of 10 miles per gallon of petrol and via Ghaziabad with an additional journey of 40 miles at the rate of 15 miles per gallon. Find the average mileage per gallon.

Ans: $\mathbf{1 0 . 5 8}$ miles per gallon.
108. The consumption of petrol by a motor was a gallon for 20 miles while going up from plains to hill station and a gallon for 24 miles while coming down. What particular average would you consider appropriate for finding the average consumption in miles per gallon for up and down journey, and why?

Ans: Harmonic Mean = $\mathbf{2 1 . 8 2}$ miles per gallon.
109. A man having to drive 90 kilometres wishes to achieve an average speed of 30 kilometres per hour. For the first half of the journey he averages only $20 \mathrm{~km} . \mathrm{p} . \mathrm{h}$. What must be his average for the second half of the journey if his overall average is to be $30 \mathrm{~km} . \mathrm{p} . \mathrm{h}$.

Ans: 60 km. p.h.
110. 35. An aeroplane travels distances of $d_{1}, d_{2}$ and $d_{3} k m s$. at speed $V_{1}, V_{2}$ and $V_{3} k m$. per hour respectively. Show that the average speed (V) is given by:

$$
\frac{d_{1}+d_{2}+d_{3}}{V}=\frac{d_{1}}{V_{1}}+\frac{d_{2}}{V_{2}}+\frac{d_{3}}{V_{3}}
$$

111. A person purchases one kilogram of cabbage from each of the four places at the rate of 20 kg ., $16 \mathrm{~kg} ., 12 \mathrm{~kg}$. and 10 kg . per 100 rupee respectively. On the average how many kg.of cabbage has he purchased per 100 rupee?
112. If you spend Rs 100 per week on apples and the price of apples for three weeks is Rs 25 , Rs 20 and Rs 10 per kilogram, what is the average price of apples for you?

Ans: Rs 15.79
113. In a certain office a letter is types by A in 4 minutes. The same letter is types by B, C and D in $5,6,10$ minutes respectively. What is the average time taken in completing one letter? How many letter do you expect to be typed in one day comprising of 8 working hours.
Ans: H.M. = 5.58 minutes per letter; Letters type in 8 hours ( 480 minutes) $=\frac{480}{5.58}=86$
114. A scooterist purchased petrol at the rate of Rs 24 , Rs 29.50 and Rs 36.85 per litre during three successive years. Calculate the average price of petrol:
(i) If he purchased 150, 180 and 195 litres in the respective years; and
(ii) If spent Rs 3,850 , Rs 4,675 and Rs 5,825 in three years

Give support to your answer.
Hints: Average price of petrol/litre $=\frac{\text { Total money spent on petrol }}{\text { Total petrol consumed in litres }}$
(i) Weighted A.M. of prices the weights being the quantities of petrol purchased.
(ii) Weighted H.M. of prices, the weights being money spent on petrol.

Ans: (i) Rs 30.65/ litre (ii) Rs 30/litre (approx.)
115. Define Arithmetic Mean, Harmonic Mean and Geometric Mean for a set of $n$ observations and state the relationship between them.
Ans: $A \geq G \geq \mathbf{H}$; the sign of equality holds if and only if all the observations are equal.
116. Show the relationship between arithmetic mean and harmonic mean for the variable X , which can take the values $a$ and $b$ such that $a, b$ are non-negative integers.

$$
\text { Ans: } \mathrm{A} \times \mathrm{H}=\left(\frac{a+b}{2}\right)\left(\frac{2 a b}{a+b}\right)=\mathrm{ab}=G^{2}
$$

117. If for two numbers, the arithmetic mean is 25 and the harmonic mean is 9 , what is the geometric mean of the series.
118. If A.M. of two numbers is 17 and G.M. is 15 , find the H.M. of these numbers.

Ans: G.M. = 15

Ans: $\mathbf{1 3 . 2 4}$
119. Comment on the following: "The G.M. and A.M. of a distribution are 27 and 30. Then H.M. is 26."

Ans: Since A.M. $\geq$ G.M. $\geq$ H.M.; the statement is correct. G.M. $=\sqrt{A . M . \times H . M .}=\sqrt{30 \times 26} \neq$ 27. So the statement is false from another angle.
UNIT - II

1. What do you mean by Range?
2. Give three different fields where range is used.
3. Give two advantages of Range.
4. Give three disadvantages of Range.
5. Give three essential features of a good measure of dispersion.
6. What do you mean by absolute measure of dispersion?
7. What do you mean by Relative measure of dispersion?
8. Distinguish between absolute and relative measure of dispersion.
9. What is the Inter Quartile Range?
10. What is Quartile Deviation?
11. Give three advantages of IQR.
12. Give three disadvantages of IQR.
13. Compare IQR with QD as measures of dispersion.
14. What is mean deviation?
15. Give three advantages of Mean Deviation.
16. Give three disadvantages of Mean Deviation.
17. What is Standard Deviation?
18. Give three advantages of Standard Deviation.
19. Give three disadvantages of Standard Deviation.
20. Compare mean deviation with standard deviation.
21. Taking three imaginary items prove that S.D. is independent of change of origin.
22. Taking three imaginary items prove that S.D. is dependent of change of scale.
23. Write three essential properties of Standard Deviation.

## PRACTICAL PROBLEMS:

1. Find the range and the coefficient of range for the following observations:

65, 70, 82, 59, 81, 76, 57, 60, 55 and 50
Ans: 32; 0.2424
2. Find the value of third quartile if the values of first quartile and quartile deviation are 104 and 18 respectively.

Ans: $\mathrm{Q}_{\mathbf{1}}=\mathbf{1 4 0}$
3. Why is standard deviation considered to be the best measure of dispersion? Find the variance if $\sum x^{2}=150$ and $\mathrm{N}=6$. Deviations are taken from actual mean.

Ans: 25
4. From the following information, find the standard deviation for X and Y variables:

$$
\sum X=235 \quad \sum Y=250 \quad \sum X^{2}=6,750 \quad \sum Y^{2}=6,840 \mathrm{~N}=10
$$

Ans: $\sigma_{x}=11.08 ; \sigma_{y}=7.68$
5. You are given the following raw sums in a statistical survey of two variables X and Y :

$$
\sum X=240 \quad \sum Y=250 \quad \sum X^{2}=6,400 \text { and } \sum Y^{2}=7,060
$$

Ten items are included in each survey. Compute Standard Deviation of X and Y variable.
Ans: $\sigma_{x}=8 ; \sigma_{y}=9$
6. State the formula for computing standard deviation of $n$ natural numbers $1,2 \ldots \ldots$. .

$$
\text { Ans: } \sigma=\sqrt{\frac{1}{12}\left(n^{2}-1\right)}
$$

7. Show that the standard deviation of the natural numbers $1,2,3,4$ and 5 is $\sqrt{2}$
8. Mean of 10 items is 50 and S.D. is 14 . Find the sum of the squares of all the items.

Ans: $\sum X^{2}=26,960$
9. The standard deviation calculated from a set of 32 observations is 5 . If the sum of the observations is 80 , what is the sum of the square of these observations?

Ans: $\sum X^{2}=1,000$
10. The mean of 200 items is 48 and their standard deviation is 3 . Find the sum and sum of squares of all the items.

Ans: 9,600; 4,62,600
11. Given: No. of observations $(\mathrm{N})=100$, Arithmetic Average $(\bar{X})=2$, Standard deviation $\left(\mathrm{s} x_{\mathrm{x}}\right)=$ 4 , find $\sum X$ and $\sum X^{2}$

Ans: $\Sigma X=200 ; \Sigma X^{2}=2,000$
12. The mean of 5 observations is 3 and variance is 2 . If three of the five observations are $1,3,5$, find the other two.

Ans: 2, 4
UNIT - III

1. What is positive correlation?
2. What is negative correlation?
3. What do you mean by simple correlation?
4. What do you mean by multiple correlation?
5. What is perfect correlation?
6. What is imperfect correlation?
7. What is perfect positive correlation?
8. What is perfect negative correlation?
9. What is partial correlation?
10. What is linear correlation?
11. What is non-linear correlation?
12. What is absence of correlation?
13. What do you mean by Standard Error in correlation?
14. What do you mean by Probable Error in correlation?
15. What do you mean by regression?
16. What is the meaning of regression lines?
17. What do you mean by regression coefficient?
18. What is standard error of estimate?
19. What is Scatter diagram?
20. What is the method of Least Square in Regression Analysis?

## Short problems:

1. In a bivariate sample, the sum of the squares of differences between the ranks of observed values of two variables is 231 and the correlation coefficient between them is -0.4 . Find the number of pairs.

Ans: $\mathrm{n}=10$
2. The coefficient of rank correlation of the marks obtained by 10 students in biology and chemistry was found to be 0.8 . It was later discovered that the difference in ranks in two subjects obtained by one of the students was wrongly taken as 7 instead of 9 . Find the correct coefficient of rank correlation.

Ans: Correct value of $\mathrm{R}=0.6061$.
3. The coefficient of rank correlation of the marks obtained by 10 students in Statistics and Accountancy was found to be 0.2 . It was later discovered that the difference in ranks in two subjects obtained by one of the students was wrongly taken as 9 instead of 7. Find the correct coefficient of rank correlation.

Ans: Correct value of $\mathrm{R}=0.3939$.
4. The coefficient of rank correlation of the marks obtained by 12 students in biology and chemistry was found to be 0.6 . It was later discovered that the difference in ranks in two subjects obtained by one of the students was wrongly taken as 8 instead of 3 . Find the correct coefficient of rank correlation.

Ans: Correct value of $\mathrm{R}=\mathbf{0 . 7 9 2 4}$.
5. Given the information: Sum of $X=5$; Sum of $Y=4$

Sum of squares of deviation from the mean of $X=40$; Sum of squares of deviation from the mean of $Y=50$; Sum of the products of deviations from the mean of $X$ and $Y=32$. Number of pairs of observations $=10$
Calculate:
(i) Regression coefficient of Y on X ; (ii) Regression coefficient of X on Y
(iii) Karl Pearson's coefficient of correlation

$$
\text { Ans: } b_{x y}=0.80 ; b_{y x}=0.64 r_{(x, Y)}=0.7156
$$

6. For some bivariate data, the following results were obtained:

Mean value of variable $\mathrm{X}=53.2$ and $\mathrm{Y}=39.5$
Regression Coefficient of Y on $\mathrm{X}=-1.5$ and of X on $\mathrm{Y}=-0.38$
What should be the most likely value of X when $\mathrm{Y}=50$ ?
Also find the coefficient of correlation between two variables.
Ans: $\mathrm{X}=53.2+(-1.5)(50-39.5)=49.21 ; r=-\sqrt{(-1.5)(-0.38)}=-\sqrt{0.57}=$ - 0.7549
7. For a particular product, the sales $(\mathrm{Y})$ and advertisement expenditure $(\mathrm{X})$ for 10 years, provide the results:
$\sum X=15, \sum Y=110, \sum X^{2}=250, \sum Y^{2}=3200, \sum X Y=400$
Find the regression line of Y on X and the estimated value of Y for $\mathrm{X}=10$
8. Calculate the correlation coefficient from the following results:
$\mathrm{N}=10, \sum X=350, \sum Y=310, \sum(X-35)^{2}=162, \sum(Y-31)^{2}=22$,
$\sum(X-35)(Y-31)=92$
Also find the regression line of Y on X .
9. For bivariate data, you are given the following:
$\sum(X-58)=46, \Sigma(Y-58)=9, \sum(X-58)^{2}=3086, \sum(Y-58)^{2}=483, \sum(X-58)(Y-$ $58)=1095$. No. of pairs of observations is 7 . You are required to determine the two regression equation and the coefficient of correlation between X and Y .

Ans: Regression Equations:
$Y$ on $X: Y=0.372 X+35.266 ; X$ on $Y: X=2.197 Y-65.680 ; r(X, Y)=0.904$ If the two regression line corresponding to two variables $X$ and $Y$ meet at a point $(2,30) V_{(X)}=$ 4, $\mathrm{V}_{(\mathrm{Y})}=1$ and correlation coefficient between X and Y is $\frac{1}{2}$, the estimated value of Y for $\mathrm{X}=$ 6 is:
10. Let the two variables X and Y have the covariance and correlation coefficient between them as 2 and 0.5 respectively and $\mathrm{V}(\mathrm{X})=2 \mathrm{~V}(\mathrm{Y})$, then the regression coefficient of X on Y is:
11. For bivariate data the mean value of $X$ is 20 and the mean value of $Y$ is 45 . The regression coefficient of Y on X is 4 and that of X on Y is $1 / 9$. Find:
(i) The coefficient of correlation.
(ii) The standard deviation of X if the standard deviation of Y is 12 .
(iii) Also write down the equation of regression lines.

Ans: (i) 0.67 (ii) $\sigma_{x}=\mathbf{2}$ (iii) Regression equations of $Y$ on $X$ and $X$ on $Y$ are respectively:

$$
Y=4 X-35 ; 9 X=Y+135
$$

12. 31. From the following results, obtaine the two regression equations and estimate the yield when the rainfall is 22 cms . and the rainfall when the yield is 600 kgs .

| Yield in kgs. | Rainfall in cms. |
| :---: | :---: |
| $(\mathrm{X})$ | $(\mathrm{Y})$ |
| 508.4 | 26.7 |
| 36.8 | 4.6 |

Coefficient of correlation between yield and rainfall is 0.52 .
[C.A. (Foundation), Nov. 2001]
Ans: $\mathrm{Y}=\mathbf{4 . 1 6 X}+\mathbf{3 9 7 . 3 2 8}: \mathbf{X}=\mathbf{0 . 0 6 5 Y} \mathbf{- 6 . 3 4 6 :} \mathbf{4 8 8 . 8 5} \mathrm{kgs} . ; \mathbf{3 2 . 6 5 4} \mathrm{cms}$.
13. The following table shows the mean and standard deviation of the prices of two shares in a stock exchange:

| Shares | Mean (in Rs) | Standard deviation (in Rs) |
| :--- | :---: | :---: |
| A Ltd. | 39.5 | 10.8 |
| B Ltd. | 47.5 | 16 |

If the coefficient of correlation between the prices of two shares is 0.42 , find the most likely price of share. A corresponding to a price of Rs 55 observed in the case of share B.

Ans: $\mathrm{X}=\mathbf{0 . 2 7} \mathrm{Y}+\mathbf{2 6 . 6 7 5}$ : Rs 41.52
14. Given the following information:

|  | X | Y |
| :--- | :---: | :---: |
| Mean | 6 | 8 |
| Standard Deviation | 5 | 13 |
| Coefficient of Determination | $=0.64$ |  |

Find: (i) $b_{y x}$ and $b_{x y}$ and (ii) Value of $Y$ when $X=100$
Ans: (i) $r^{2}=0.64 \Rightarrow r= \pm 0.8 ; b_{y x}=2.08$ or -2.08 and $b_{x y}=0.31$ or -0.31 (ii) 203.52

## Regression Analysis:

1. What do you mean by regression?
2. What is regression coefficient?
3. What is regression line?
4. What is coefficient of determination?
5. What is scatter diagram?
6. Explain the method of least square in regression.

## Practical problems:

1. If the two regression line corresponding to two variables $X$ and $Y$ meet at a point $(2,30)$ $V_{(X)}=4, V_{(Y)}=1$ and correlation coefficient between $X$ and $Y$ is $\frac{1}{2}$, find the estimated value of $Y$ for $X=6$
2. Let the two variables $X$ and $Y$ have the covariance and correlation coefficient between them as 2 and 0.5 respectively and $V_{(X)}=2 V_{(Y)}$, then find the regression coefficient of $X$ on Y.
3. For bivariate data the mean value of $X$ is 20 and the mean value of $Y$ is 45 . The regression coefficient of $Y$ on $X$ is 4 and that of $X$ on $Y$ is $1 / 9$. Find the coefficient of correlation.
4. For bivariate data the mean value of $X$ is 20 and the mean value of $Y$ is 45 . The regression coefficient of $Y$ on $X$ is 4 and that of $X$ on $Y$ is $1 / 9$. Find the standard deviation of $X$ if the standard deviation of Y is 12.
5. For bivariate data the mean value of $X$ is 20 and the mean value of $Y$ is 45 . The regression coefficient of $Y$ on $X$ is 4 and that of $X$ on $Y$ is $1 / 9$. Write down the equation of regression lines.

> UNIT - IV

Index Numbers:

1. What do you mean by index numbers?
2. Give the purposes for which index number is constructed.
3. What is Price Index Number?
4. What is Quantity Index Number?
5. What is Value Index Number?
6. What do you mean by change of base of index numbers?
7. What do you mean by splicing of index numbers?
8. What is deflating of index numbers?
9. What is cost of living index number?
10. What is a base year?
11. What is Time Reversal Test?
12. What do you mean by Factor Reversal Test?
13. What is Circular Test?
14. What do you mean by Unit Test?
15. Explain unweighted index numbers.
16. When is weighted index number used?
17. Explain Laspeyre's method of index number.
18. Explain Paasche's method of index number.
19. Why is Fisher's method called Ideal Index number?

Practical Problems:

1. The geometric mean of index number of Laspeyre and Paasche is 229.5648 while the sum of Laspeyre's and Paasche's index number is 480. Find out Laspeyre's and Paasche's indices.

Ans: 310; 170
2. A company spent Rs 50 , Rs 48 , Rs 18 and Rs 42 during 1998. The company increased the expenditure to Rs 100 , Rs 98 , Rs 60 and Rs 102 respectively on four commodities. If the units of four commodities purchased during 1998 and 1999 are identical i.e. 5, 2, 6 and 17, compute the price index for 1999 by the most suitable method.

Ans: Simple Aggregative method 227.85
3. Given that $\sum \mathrm{p}_{1} \mathrm{q}_{1}=250, \sum \mathrm{p}_{0} \mathrm{q}_{0}=150$. Paasche's Index Number $=150$ and Dorbish-Bowley's Index Number = 145, find out (i) Fisher's Ideal Index Number; and (ii) Marshall-Edgeworth's Index Number.

Ans: 144.9, 145.26
4. A worker earned Rs 900 per month in 2010. The cost of living index increased by $70 \%$ between 2010 and 2013. How much extra income should the worker have earned in 2013 so that he could buy the same quantities as in 2010 ?

$$
\text { Ans: } \operatorname{Rs} 12 \times\left[\left(\frac{170}{100} \times 900\right)-900\right]=\text { Rs 7,560 }
$$

5. During a certain period the cost of living index number goes up from 110 to 200 and the salary of the worker is also increased from Rs 325 to Rs 550. Does the worker really gain, and if so, by how much in real terms?

Ans: Loss of Rs 90.90
6. In calculating the cost of living index the following weights were used: Food $8 \frac{1}{2}$; Rent 2; Clothing $2 \frac{1}{2}$; Fuel and Light 1; Miscellaneous 11. Calculate the index number for a data when the percentage increase in prices of the various items over prices of July, 2018 = 100 were $31,57,90$, 75 and 88 respectively.

Ans: 152.2
7. In calculating a certain cost of living index number, the following weights were used. Food 15, Clothing 3, Rent 4, Fuel and Light 2, Miscellaneous 11. Calculate the index for a date when the average percentage increases in price of item in the various groups of the base period were 32 , $54,47,78$ and 58 respectively.

Suppose a business executive was earning Rs 2,050 in the base period. What should be his salary in the current period if his standard of living is to remain the same?

Ans: 141.76; Rs 2,906.08
8. The cost of living index uses the following weights:

Food 40, Rent 15, Clothing 10, Fuel 10, Miscellaneous 15. During the period $2010-15$, the cost of living index raised from 100 to 205.83 . Over the same period the percentage rise in prices were:

Rent 60, Clothing 180, Fuel 75 and Miscellaneous 165. What is the percentage of change in the price of food?

Ans: 90
9. The relative importance of the following eight groups of family expenditure we found to be Food 348, Rent 88, Clothing 97, Fuel and Light 65, House durable goods 71, Miscellaneous goods 35, Services 79, Drink and tobacco 217. The corresponding \% increase in price for Oct. 2015 gave the following values $-25,1,22,18,14,13$, ?and 4 . Calculate the percentage increase in group services, if the percentage increase for whole group is 15.278.

Ans: 11
10. From some given data, the retail price index based on five items, viz. Food, Rent and Rates, Fuel and Light, Clothing and Miscellaneous was calculated as 205. Percentage increases in prices over the base period are given below:

Rent and Rates 60, Clothing 210, Fuel and Light 120, Miscellaneous 130
Calculate the percentage increase in the Food Group, given that the weights of different items are as follows:
Food 60, Rent and Rates 16, Fuel and Light 8, Clothing 12, Miscellaneous 4, All items 100.
Ans: 92.3\% increase in food group

1. What do you mean by residual method?
2. What is meant by Direct Percentage method?
3. What is Harmonic analysis method?
4. What is meant by reference cycle analysis method?
5. Explain the meaning of secular trend?
6. Write short note on seasonal variation.
7. What do you mean by cyclical variation?
8. What is erratic variation?

## Practical Problems:

1. In a study of its sales, a motor company obtained the following least square trend equations: $Y=1,600+200 \mathrm{X}$ (origin 2010, X units $=1$ year; $\mathrm{Y}=$ total number of units sold annually) The company has physical facilities to produce only 3,600 units a year and it believes that it is reasonable to assume that at least for the next decade the trend will continue as before.
(a) What is the average annual increase in the number of units sold?
(b) By what year will the company's expected sales have equaled its present physical capacity?
(c) Estimate the annual sales for 2015.

How much in excess of company's present physical capacity is this estimated value?
Ans: (a) 200 units (b) In 2020 (c) 4,600 units: Excess $=\mathbf{4 , 6 0 0}-\mathbf{3 , 6 0 0}=\mathbf{1 , 0 0 0}$ units
2. Convert the following annual trend equation for total sales of a company to a monthly trend equation:

$$
Y=162+15.8 X \text { (Origin: 2015; Scale: } 1 \text { units of } X=1 \text { year })
$$

Forecast the sales for June, 2018 by the two equations. Compare your results.
Ans: $Y=13.5+0.1097 X$; (Origin: 2015, $X$ unit $=1$ month: $Y$ unit $=$ monthly sales)
3. The trend of the annual sales of Bharat Aluminum Company is described by the following equation:
$\mathrm{Y}_{\mathrm{c}}=12+0.7 \mathrm{X}:($ Origin: 2010; X unit $=1$ year and Y unit $=$ Annual production $)$
Step the equation down to a month to month basis and shift the origin to $1^{\text {st }}$ January 2010.

> Ans: $\quad Y_{c}=1+\frac{0.7}{144} X ;\left(\right.$ Origin: $1^{\text {st }}$ July 2010; X unit $=1$ month $) ;$ $Y_{c}=0.9712+0.0048 X$ (Origin: $1^{\text {st }}$ January, 2010)
4. The trend equation for certain production is given by: $\mathrm{Y}=3,600+288 \mathrm{t}$; where
$\mathrm{Y}=$ Annual production in thousand tons: t : Time with origin, the year 2010 and unit $=$ 1 year
Estimate the trend value of the production for September, 2014.
[I.C.W.A (Intermediate) June, 2000]
Hints: Monthly trend equation is given by:
5. $\mathrm{Y}=\frac{3,600}{12}+\frac{288}{144} \mathrm{t}=300+2 \mathrm{t}$ : Origin: $1^{\text {st }}$ July, 2010; t: Unit 1 month; Y : Monthly production For September 2014 i.e. $15^{\text {th }}$ September, 2014: $t=4 \times 12+2.5=50.5$ Estimated production for $2014=300+2 \times 50.5=401$ thousand tons.

